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ABSTRACT

This document presents ideas and activities for teaching algebra. The section on "Week by Week Essentials" provides seven resources in a weekly format. It includes writing ideas that provide an algebra prompt and requires students to organize their thoughts and present them in a coherent fashion, and connections to the real world that identify situations or problems where algebra is an important tool in their investigations and explanations. Also included are specific mathematics vocabulary; ideas from teachers about organization, management, assessment, curriculum, standards, projects, and grading; calculator tips that identify the calculator routines that are most likely to be used; problems that review a recently covered concept or skill; and problems for students to do outside of class that further extend concepts or skills students have used in the past. The "Activities" section contains detailed layouts for classroom exercises designed to last for an hour or more. Supporting materials are provided in the "Blackline Masters." (JRH)

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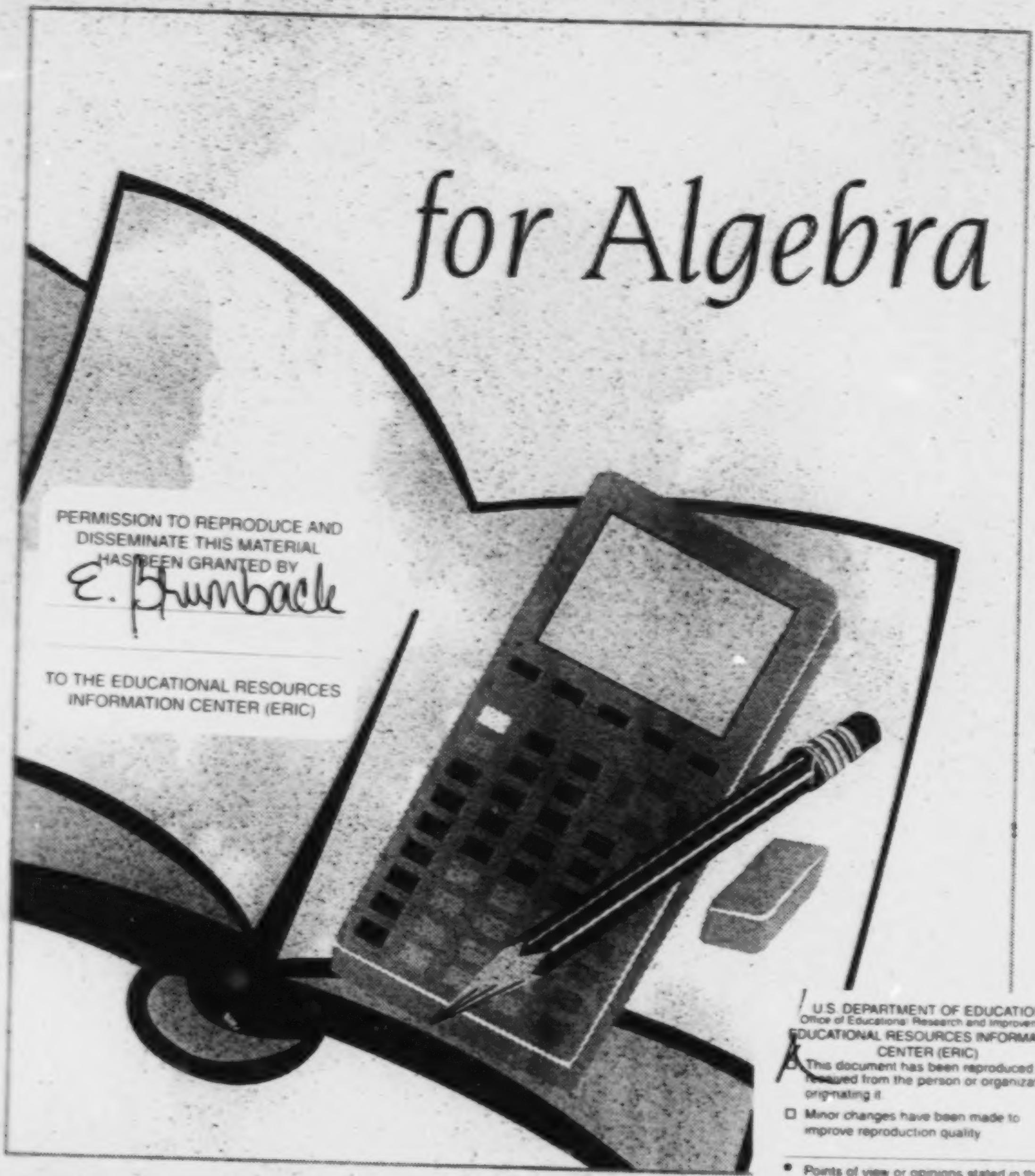
Resources

for Algebra



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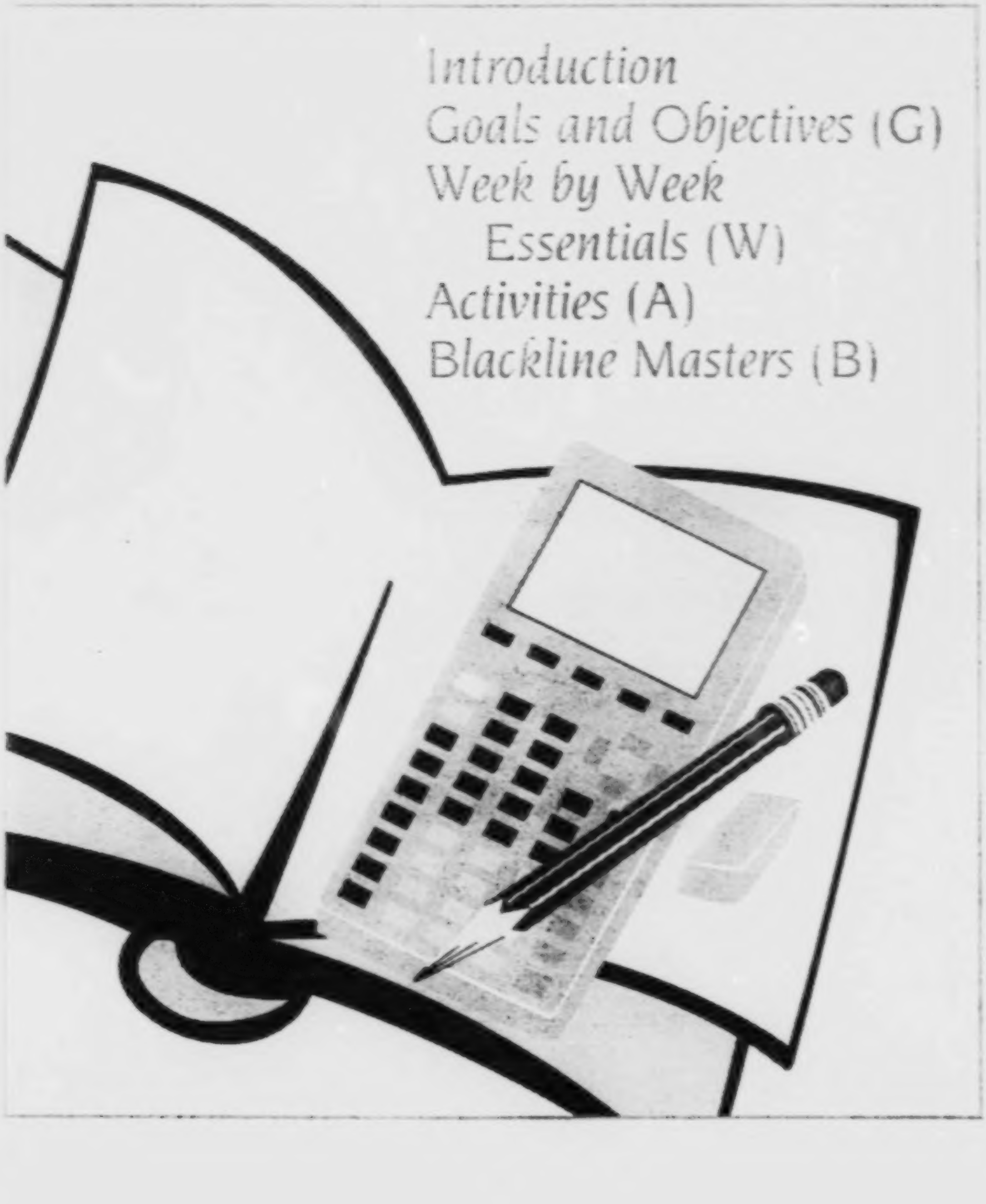
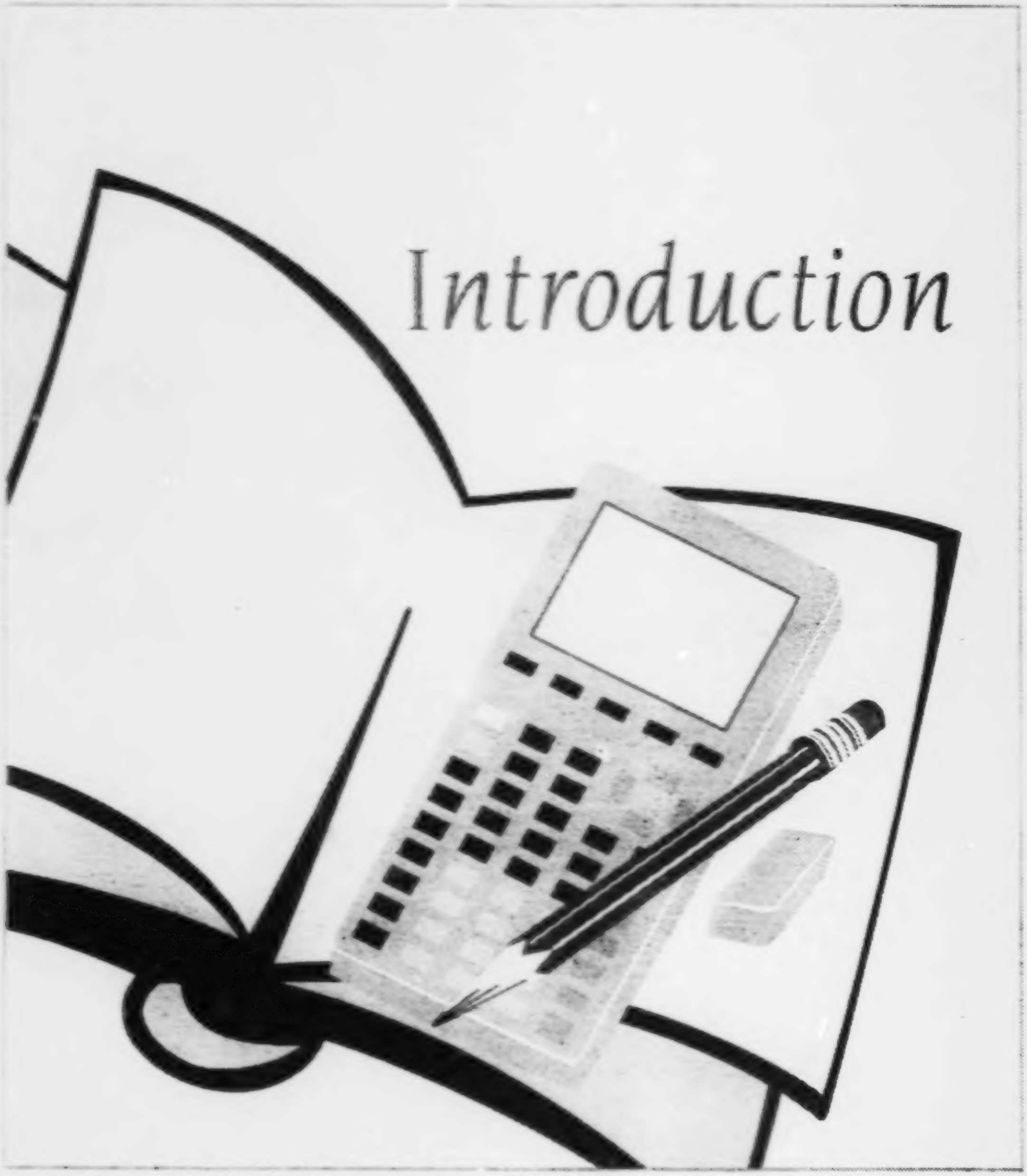


Introduction



Introduction

★

Algebra is an universal theme that runs through all of mathematics and is important in nearly every aspect of the workplace. Algebraic thinking is a necessity for the intelligent consumer of goods and information.

★ ★ ★ ★

In 1989 algebraic ideas became a strand that runs through the **K-8** curriculum (see **B-12** and **B-13**). In 1991 the North Carolina State Board of Education designated **Algebra 1** as one of three units of mathematics required for high school graduation. With curriculum revisions approaching, the Department of Public Instruction began five years of staff development for Algebra teachers. Over one thousand teachers participated in at least thirty hours of workshops. The ***Resources for Algebra*** document is offered as a culmination to that staff development and an opportunity for teachers to re-examine the 1992 **Algebra 1 Standard Course of Study**.

★ ★ ★ ★ ★ ★ ★

Resources for Algebra presents **Algebra 1**, wherever possible, as **calculator-assisted** and **activity-** or **application-centered**. Teachers should feel free to modify, reorder, and/or rewrite any or all of the ideas and activities. This is a document that teachers can build upon and remake to fit their classroom. It is a good model for a staff development activity within a school or school district. Using a similar format, teachers can create their own entries and share their resources in a similar document.

Problems and activities are correlated with the **Algebra Standard Course of Study** in the ***Goals and Objectives*** **(G-1 - G-68)** section.

* * *

Week by Week Essentials **(W-1 - W-80)** provide seven resources in a weekly format. Although they are presented in chronological order, teachers should review the entire section before jumping in. The second page of each ***Week*** is in larger type so that it can easily be copied for use as a transparency. ***Writing Ideas*** provides an algebra prompt. Writing in the context of algebra requires students to organize their thoughts and to present them in a coherent fashion. It allows students the opportunity to reflect, persuade, and/or report mathematics for themselves as well as for others. ***Connections to the World*** identifies situations or problems where algebra is an important tool in their investigation and explanation. ***The WORDS are ...*** is intended to highlight and encourage clear, specific mathematics vocabulary in the classroom and among colleagues. ***Teacher to Teacher*** are ideas from colleagues about organization, management, assessment, curriculum, standards, projects, and grading. ***Calculator Tips*** tries to identify the calculator routines most likely to be used in an **Algebra 1** classroom. Since these were written from a TI-81/82 perspective, teachers that use other models or brands should construct similar routines for their calculators.

* * *

Warm Ups are problems that review a recently covered concept or skill and can be solved by most students in ten minutes or less. (See **W-73 - W-80** for answers.) ***Challenges*** are problems for students to do outside of class and extend concepts or skills students have used or are experienced with.

The ***Activities*** **(A-1 - A-48)** are detailed layouts (concepts and competencies, goal, materials, description, and procedure) for classroom exercises which usually extend for a hour or more. Although they are identified in the ***Goals and Objectives*** by objective, the ***Activities***, like many other entries, address several objectives. Many of the supporting materials for the ***Activities, Goals and Objectives,*** and ***Week by Week Essentials*** are provided in the ***Blackline Masters*** **(B-1 - B-143)**.

* * *

Resources for Algebra has been in progress for the last two years. We greatly appreciate the time spent writing, editing, and revising ideas and activities by the principle writers, Becky Caison, Ann Crawford, and Jan Wessell. Many others contributed also, including Pat Baker, Eddie Banks, Kay Bennett, Betty Davis, Cyndy Davis, Susan Foster, Jennifer Gravely, Barbara Hardison, Kathy Hodge, Marcia Isley, Liz Kimbro, Julie Kolb, Sandra Leary, Lynne Lewis, Ginny Maxwell, Sarah Melton, Kathy Moore, Judy Mungle, Helen Owen, Debra Parker, Debbie Pergerson, Elfreda Robinson, P.A. Trogdon, Elsie Whisenant, and the NCDPI staff. Mathematics teachers in Richmond and Union counties provided valuable feedback.

After you have had a chance to review and use these materials, please take a moment to let us know if ***Resources for Algebra*** has been useful to you. Your evaluation is important for the Eisenhower project that supported the development of these materials. Please respond to:

Bill Scott
Mathematics and Science Section
Department of Public Instruction
301 N. Wilmington Street
Raleigh, NC 27601-2825

Indicate the extent to which you agree with statements 1-5.

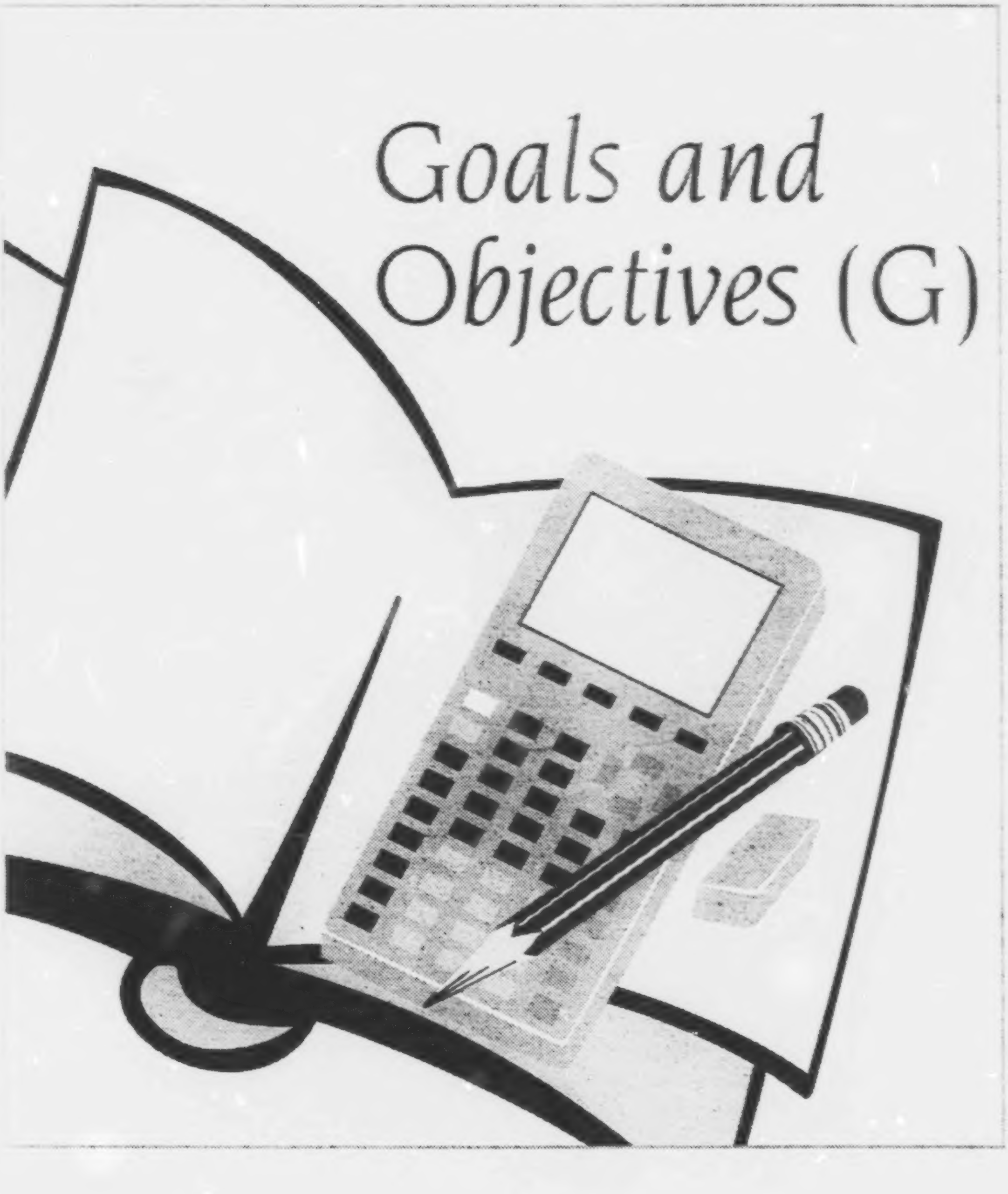
	Strongly Disagree				Strongly Agree
1. The materials will be helpful in teaching the goals and objectives for Algebra I in the Standard Course of Study.	1	2	3	4	5
2. I plan to use these materials with my students in (course) _____________ .	1	2	3	4	5
3. The materials are appropriate for Algebra I.	1	2	3	4	5
4. The materials are interesting and engaging.	1	2	3	4	5
5. The format of the materials encourages their use.	1	2	3	4	5

(over)

6. How do you plan to use these materials in your classroom?

7. Additional comments:

Goals and Objectives (G)



The learner will use the language of algebra.

1

1.1 Evaluate algebraic expressions.

A. Use your graphing calculator to store values for variables using the **STO** function. Evaluate expressions with the calculator.

B. Give the students a set of expressions on the board (the overhead). Roll a die to determine the value to substitute for each expression.

C. **Calculator Tips (W-1, 41)**

D. **Warm Ups (W-4, 10, 12, 40, 68, 72)**

1.2 Use formulas to solve problems.

A. Ask the students to talk to their parents and others to find out what kinds of formulas they use in life. In groups during class compile a list of the formulas used. Match the formula to who uses it. Have each group design a poster to illustrate the formula. Solve for different variables in the formula.

B. Hand out formulas on index cards. Ask the students to write a problem for the formula he/she receives. Switch cards and repeat the process.

C. The formula to convert Celsius (°C) to Fahrenheit (°F) is **$F = (9/5)C + 32$**. Derive the formula to convert Fahrenheit to Celsius. Have the students graph the formula on the calculator and create a table to convert between the two temperature scales.

D. Sports offer a particularly rich context for using formulas. Let students investigate formulas in sports where they are most interested and create problems based on those formulas. Here is an example.

According to the NCAA, Maseo Bolin, NC A&T State University, was the highest rated quarterback in North Carolina during the 1995 football season. The NCAA uses the following formula to measure the passing efficiency of quarterbacks.

(yards per attempt) • 8.4 + completion percentage + touchdown percentage • 3.3 - interception percentage • 2

For Bolin that would be:
$(2276/299) \cdot 8.4 + (100 \cdot 162/299) + (100 \cdot 17/299) \cdot 3.3 - (100 \cdot 5/299) \cdot 2 = 133.5$

Find the appropriate statistics for the college quarterbacks in North Carolina from the most recent football season. Rank them using the formula for passing efficiency.

E. **Connections to the World (W-1, 5, 13, 17)**

F. **Warm Ups (W-10, 12, 18, 20, 24, 28, 30, 32, 36, 48, 50, 60, 62, 64, 66, 70)**

G. **Challenges (W-36, 68)**

1.3 Translate word phrases and sentences into expressions and equations and vice versa.

A. Have each student write a numeric expression (phrase or sentence) in words on an index card. Switch cards with a partner. On a separate card, write the expression in symbols and numerals so that each of the original cards have a match. Check and recheck to make sure the cards have accurate matches. Take up the cards, shuffle them, and pass the cards back out to the students. Have the students again look for matches. (You may also take up the cards in small groups.)

B. Have each student select a starting number. Then: add 5, double the result, subtract 4, divide by 2, and subtract the number with which you started. The result should be 3. Use counters or blocks to show how the result is always 3. Let a particular color of counter or shape of block represent an undetermined number. Show how the result is always 3. **(B-29)** Use algebraic notation to show why the result is always 3. Try several other tricks, first illustrating with counters or blocks and then with algebraic notation. Have students create their own tricks and share with the class. Use the student creations as starter problems, classwork, homework assignments, or even test items. Be sure that the algebraic representation and explanation is a part of the assignment.

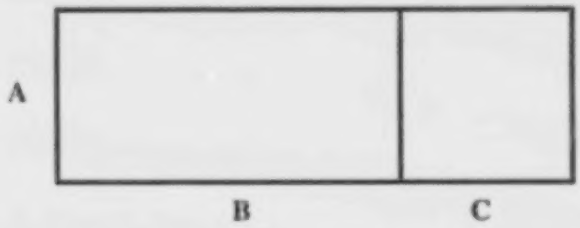
C. After trying **Basketball: With the game on the line ... (A-27)**, try **Basketball Extension 2 (B-34)**. The Basketball activities generally connect probability and statistics with algebra.

D. **Connections to the World (W-7, 9, 39, 45, 53, 55, 63)**

E. **Warm Ups (W-6, 8, 10, 16, 18, 20, 30, 34, 60, 72)**

1.4 Use the associative, commutative, and distributive properties.

A. Demonstrate the distributive property by showing it geometrically.



The area of the rectangle above can be found by finding A (B + C) or by finding (A • B) + (A • C). Show that the same can be done in other situations.

B. Use students to demonstrate the commutative and associative properties.

C. Ask the students to think of examples in real life that demonstrate a property, such as, can you tell if a person put his shirt or his pants on first? Can you tell if someone puts their socks on first or their shoes on first ... etc.

D. **Lining Up Dominoes (A-1)**

Students will make a train of dominoes by successfully applying the distributive property. Blank domino sheets (**B-99**) can be made available so that students can create versions of the game that practice various algebraic skills throughout the year. The format used in this activity can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

E. **Property Game**

On 8.5 by 11 inch sheets of colored paper, write one symbol or letter on a sheet.

(, a, b, c, =, 1,) , a, b, c, ab, ac, +, +, 0, +, +, -a, -1, 1/a, (,) , 0

Make two sets. Divide class into two groups. Have groups stand on opposite sides of the room facing you. Give each group a set of symbols/letters to distribute among the group (several students may have two sheets). Call out the name of a property. Students line up to make the property with their symbols/letters.

The distributive property would look like this:

a	(	b	+	c	)	=	ab	+	ac

The multiplicative inverse would look like this:

a	1/a	=	1

F. **Warm Ups (W-8, 28, 34, 38)**

Essentials for Instruction	Establish a daily routine; explain changes in the routine. Explain the textbook format at the beginning of the course. Provide a list of materials needed.

The learner will perform operations with real numbers

2

2.1 Simplify real number expressions with and without a calculator.

A. Play **Krypto (B-36)** with the students. A deck of Krypto cards are described as follows: three of each number 1 - 10; two of each number 11 - 17; and one of each number 18 - 25. Randomly select five cards. All five of these numbers will be combined, using any operations (including exponents and roots), to make an expression equal to your target number, a sixth card chosen randomly. With multiple sets of cards, groups of students can play.

Example: Suppose 23 was the target and 7, 8, 4, 3, and 16 were the numbers to use to reach the target.

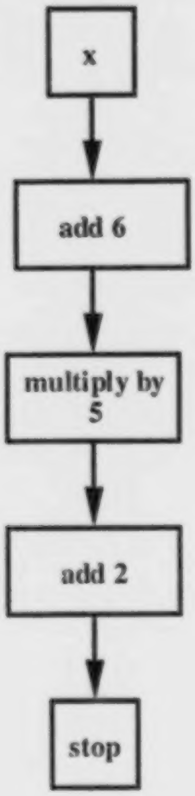
$$23 = (16 + 4 + 3) \cdot (8 - 7)$$

or

$$23 = 8 \cdot 3 - (\sqrt{16} \div 4)^7$$

B. From a standard deck of cards use the 2-10 of hearts and spades and one joker to play **What's Your Sign?** Let the red cards represent negative integers, the black cards positive integers, and the joker zero. From a second set of operation cards, created earlier by the students or teacher (one symbol or phrase per card: +, -, •, ÷, <, >, "has the same absolute value"), one is selected. Playing in groups of two, one player will shuffle and deal the 19 cards. The player with ten cards lays a card on the table and picks one from the other player's hand. Depending upon the operation card selected, the first player completes the indicated operation or answers true or false. If correct, the player discards. If incorrect, the player keeps them. The other player then takes a turn. The winner is the first player to discard all of his cards.

C. Develop or have the students develop flowcharts to simplify expressions.
Example: Choose **x**.

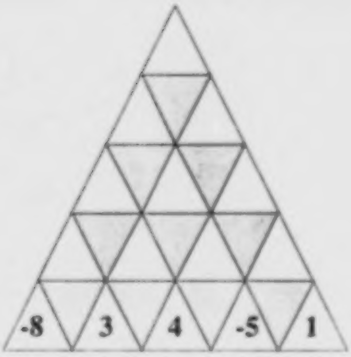


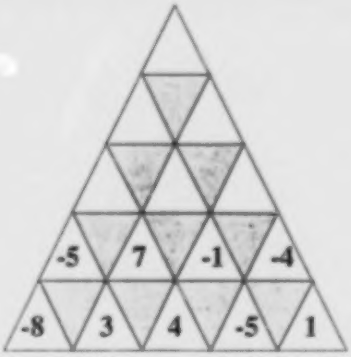
Write the expression and solution.

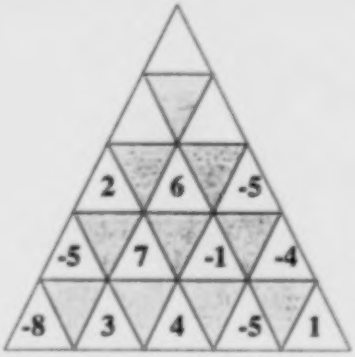
D. Roll seven dice (five WHITE, one RED, and one GREEN). The outcomes of the WHITE dice must correctly combine, using any operations, to equal 10 • GREEN + RED.
Example: GREEN = 4, RED = 6, WHITEs = 6, 4, 4, 3, 1.
One solution is 46 = 6 • (4 + 3) + 4 • 1

E. **Race to the Top (B-37, 38, 39)**

Enter five numbers in the small triangles along the base of the large triangle. To fill in space in the row above, carry out a teacher-specified operation on the numbers in the two spaces immediately below. The example below uses addition.







F. **Four in a Row**

Students will need game boards **(B-40)**, markers of two different shapes or colors, and two paper clips. Play begins with the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another.

G. **Lining Up Dominoes (A-1)**

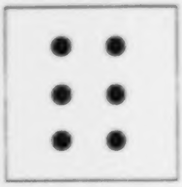
Students will make a train of dominoes by successfully simplifying an expression. Blank domino sheets **(B-99)** can be made available so that students can create versions of the game that practice various algebraic skills throughout the year.

H. **Relays (B-41, 42)**

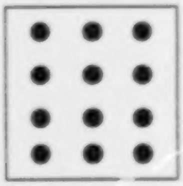
Students will work in teams. Individually team members will complete a problem and share the result with the rest of the team so that a team task can be finished. The format used in this activity can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

1. **Multiplying Integers Using Counters**

Select two different colors of counters for the overhead and decide which color represents positive and which represents negative (red = negative and blue = positive). Demonstrate the concept of multiplying a positive times a positive.
For example, 3 • 2.
Interpret multiplying a positive times a positive as "putting in" counters.
3 • 2 will mean "putting in" 3 groups of 2 positive counters each.

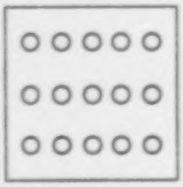


Have students illustrate 4 • 3.



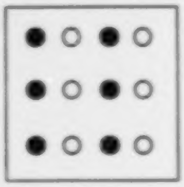
Have the students illustrate other examples.

Illustrate 3 • -5 as putting in 3 groups of 5 negative counters.

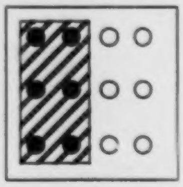


Have students illustrate other problems such as 2 • -4 and 4 • -1.

Since positive means to "put in" then a negative means to "take out".
-3 • 2 means to "take out" 3 groups of positive 2. First start with zeros. A positive and a negative counter represent zero. Since 3 • 2 = 6, we need at least 6 zeros.

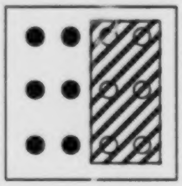


Remove 3 groups of positive 2. You are left with -6.



Have students illustrate -2 • 4. You need 8 zeros. Remove 2 groups of positive 4. You are left with -8. Students should illustrate several other examples.

To illustrate -3 • -2 we need to remove 3 groups of negative 2. Again start with 6 zeros as in the previous example. Remove 3 groups of negative 2. You are left with positive 6.



Have students illustrate -4 • -1 and -2 • -5.

J. **I Have ... Who Has ... " (A-3)**
Students will listen, perform operations, and respond when appropriate in a round-robin format. Students will need to be able to complete operations with integers using paper and pencil. Students will use the format to create their own versions of the activity. The format used in this activity can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

K. **Warm Ups (W-2, 4, 6, 8, 14, 22, 26, 30, 32, 34, 36, 38, 40, 42, 44, 46, 50, 52, 60, 62, 66, 68, 72)**

L. **Challenges (W-2, 6, 8, 12, 14, 16, 18, 20, 26, 28, 32, 38, 42, 44, 50, 64)**

2.2 Determine the additive or multiplicative inverse of a number.

A. Give each student a chalkboard. Call out a number. Ask the students to write the additive inverse on the chalkboard. Wait a few seconds and ask for the students to show their answers. Monitor the students. Make notes of students who need help. Use the same process with multiplicative inverses.

B. Do the same kind of activity as above using cards (ask the students to make number cards on index cards). Write a number on the card. Write the additive (or multiplicative) inverse on another card. Shuffle the cards. Have students in a group match the inverses.

C. **Warm Ups (W-26, 58)**

2.3 Determine the absolute value of expressions.

A. In groups, or with partners, ask the students to write a flowchart to determine the absolute value of any expression.

B. Brainstorm problem situations where absolute value is needed. Write problems using absolute value. Put the problems on a sheet and give the students the problems as a homework assignment.

C. **Absolutely !! (B-43)**
The object of the game is to cover as many of the integers -6 to 6 on the game board as possible and accumulate the lowest score possible. The score is the sum of the absolute values of the uncovered integers. Counters, to cover the integers on the board, and a pair of dice (different colors) are needed for each pair of players.

- Roll the dice (one is positive, one is negative).
- The player then covers the integers on the board that correspond to exactly one of the following: (1) the value on either dice, not both, (2) the value of the sum, or (3) any two values having the same sum as the sum just rolled.
- A player rolls until he is unable to cover an integer. That player sums up the absolute values of the uncovered integers and records for round 1.
- The other player then rolls.
- After five rounds are played, the players total the rounds and the player with the lowest score wins.

Example: Two dice, green (positive) and red (negative). Roll. 2 and -3. Cover -3. Roll. 3 and -1. Cover 3. Roll. 1 and -4. Cover -4. Roll. 1 and -2. Cover -2. Roll. 5 and -6. Cover -6. Roll. 2 and -1. Cover 2. Roll. 3 and -6. End of Round 1 for this player. 3, -6, -3 (3 + -6) are covered and there are no remaining pairs that will combine to make -3. This player's score is 22, the sum of the absolute values of the uncovered integers (-5, -1, 0, 1, 4, 5, 6).

D. Use the number line **(B-19)** to define absolute value as the distance from zero.

E. **Calculator Tips (W-9)**

2.4 Raise a real number to an indicated power.

A. Experiment with raising a number to a power using a calculator.

B. Play match game with cards matching the expansion or answer with the number raised to a power

$$\boxed{5^2} = \boxed{5 \cdot 5} = \boxed{25}$$

$$\boxed{n^3} = \boxed{n \bullet n \bullet n}$$

C. Use clear color chips on the overhead projector. A blue chip means to multiply by 5. A red chip means to multiply by 2. On the overhead place two blue chips. Ask for the answer and the expansion. Add two red chips. Ask for the expansion and solution. Repeat the exercise changing the meaning of the chips. Example:

Ⓑ Ⓑ Ⓡ Ⓡ $= B^2R^2$

$= 25 \cdot 4$
$= 100$

D. Have students compare which is largest? 100^4 1000^3 10000^2

E. **Patterns with Exponents (B-44, 45)**
This problem set explores patterns generated when numbers are exponentially increased.

F. **Calculator Tips (W-3, 11)**

G. **Challenges (W-4, 10, 22, 24)**

H **Warm Ups (W-6, 8, 10, 16, 32, 44, 48, 54, 58, 62, 64, 70)**

I. **Connections to the World (W-19)**

2.5 Write numbers in scientific notation and use this notation with the calculator.

A. Ask the students to find numbers in newspaper articles. Bring in the article. Discuss how the numbers are used. List the numbers on the board. Write them in scientific notation.

B. Raise numbers to powers on the calculator. Write the scientific notation for the number.

C. Students can experience how scientific notation can ease calculations with large and small numbers with **Number Crunching With Ease (B-46, 47)**

D. **Scientifico (A-5)**
Students practice translating scientific notation numbers into standard notation. Students take turns rolling three dice and constructing a number in scientific notation. Ex: 3, 6, 4 can be written $3.6 \bullet 10^4$. After recording this number on the recording chart, the student places a marker in the proper place on the game board. The student who can make three numbers in a row, column, or diagonal is the winner.

E. Have students find large and small numbers in an almanac or from the Internet. Make a bulletin board with the numbers in scientific notation. Here are a few examples:

Michael Jordan's salary	43.9 million
World Population	5,607,000,000
How fast human hair grows	10^{-8} miles per hour
Tons of trash Americans throw away each yr.	70,000,000
Proton radius	1.6×10^{-13}

F. **Calculator Tips (W-5)**

G. **Connections to the World (W-11)**

H. **Warm Ups (W-46, 62)**

2.6 Distinguish between rational and irrational numbers.

A. Give each student a rational or irrational number on a card. Have the students classify their numbers and place the cards on the board in the correct category.

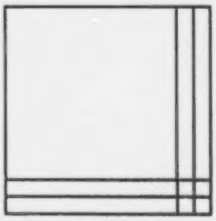
These are rational.	These are NOT rational.
$\sqrt{25}$	e
32	$\sqrt{5}$
-6	π

B. Give the students two index cards. Ask them to write an "I" on one card for irrational and an "R" on the other for rational. Show the students a rational number and ask them to silently show you which type of number it is by showing the correct card. Be very specific about when to show your answer. Ask for everyone to "show" at the same time. Note the students who consistently answer incorrectly so that you can follow up with those students.

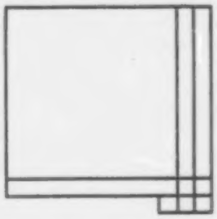
C. **Warm Ups (W-52)**

2.7 Find approximations for square roots with and without a calculator.

A. Use base ten blocks to show the students that the square root is one "side" of a square.



144 = 12 • 12. Perfect squares make a square.



You can approximate the square root by finding a "close" square.
$135 = 11 \cdot 12 + 3$

B. Use the calculator to find square roots. Round your answer appropriately. Discuss problem situations where square roots are used.

C. **Getting to the Root of the Number (A-37)**
Working in pairs, students will use base-10 blocks to build incomplete squares that represent an approximate value for a specified irrational number.

D. **Warm Ups (W-20, 30, 50)**

2.8 Simplify radical expressions.

A. Use the puzzle formats of **How Do They Fit? (A-11), Lining Up Dominoes (A-1), and "I Have ... Who Has ..." (A-3)** to create puzzles for student use. Whenever possible, let students create the puzzles.

B. **Warm Ups (W-54, 56, 60)**

2.9 Multiply two binomials which contain square roots.

A. Use area models to explain binomial multiplication. **B-74, 75, 76, 77, 78, 79.**

B. **How Do They Fit? (A-11)**
Students will assemble a $3 \cdot 3$ array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well-constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter.

2.10 Compare real number expressions.

A. Use the quantitative comparison rules from the SAT. The answer is

A if the quantity A is greater than B

B if the quantity B is greater than A

C if the two quantities are equal

D if the relationship cannot be determined with the information given

Ask the students to write the letters A, B, C, and D on index cards. Show the students two quantities like

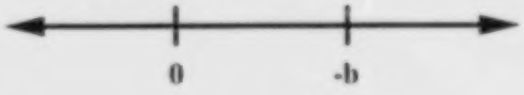
G	F
$1/n$	n

Ask the students to decide how the quantities are related by showing a card. Be sure you ask them to "show" at the same time. Give "think time". Note the students who consistently answer incorrectly. Some examples are

	A	B
1.	$4x^2$	$4(x^2)$
2.	$3x + 4^2$	$(3x + 4)^2$
3.	$x + y$	xy
4.	$r + s$	$r - s$
5.	x/y	xy
6.	$m + 1/m$	2

B. **Bull's Eye (B-48)**
The game is played with two or more people in which each player tries to reach a specified goal number with the least number of rolls of a pair of dice. Decide who goes first. If there are more than two people playing proceed in a clockwise manner. The teacher assigns the goal number. On each student's turn the student will: roll the dice and compute the sum (difference, product, quotient, powers, roots) of the two numbers, either add or subtract that result from his/her cumulative total, and record the proceeds on the score sheet. The winner is the person who reaches the goal number with the fewest rolls of the dice. If no one reaches the goal number after 16 rounds, the winner is the student who is closest to the goal number. Calculators may be reserved to use only when powers and roots are the operations of choice.

C. Place the following number line on the board:



Pair students and ask them to write down two things they know about -b and b. Next, have students write down anything they don't know about -b and b. Have students share their responses.
Ask students to discuss where the following are located on the number line:
b $(-b + -b)$ $2b$ $(-b + b)$ $-3b$ b^2

Ask students where **$(-b + 2)$** and **$(-b - 2)$** might be located on the number line

D. **Warm Ups (W-6, 64)**

E. **Challenges (W-24, 30, 34)**

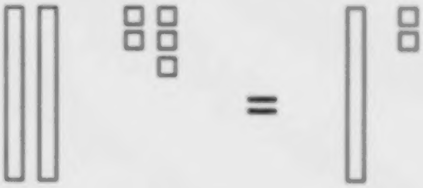
Essentials for Instruction	Have frequent review. Summarize and allow for questions. Circulate and assist. Use diagrams and other visual aides.

The learner will solve equations and inequalities with one variable.

3

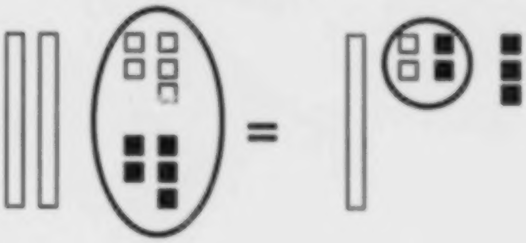
3.1 Solve a simple equation using the addition property of equality and the idea of additive inverse.

A. Use algebra tiles to demonstrate solving simple equations using the additive inverse. Let students use tiles to solve similar problems generated by the teacher or textbook. For example:

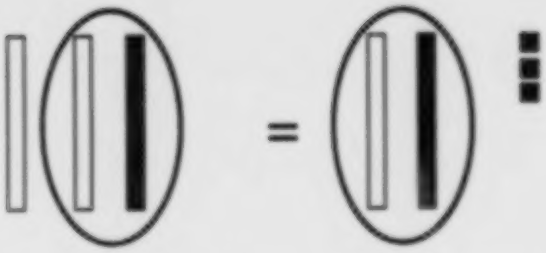


$2x + 5 = x + 2$

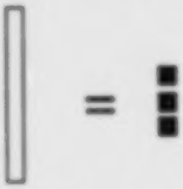
Add -5 to each side of the equation and simplify.



$2x = x + -3$



Add -x to each side of the equation and simplify.

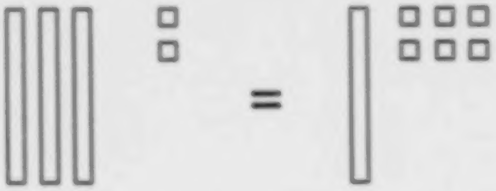


x = -3

B. **Warm Ups (W-28)**

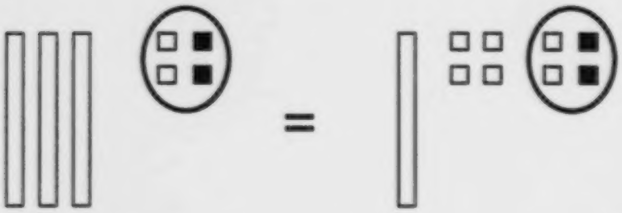
3.2 Solve a simple equation using the multiplicative property of equality and the idea of multiplicative inverse.

A. Use algebra tiles to demonstrate solving simple equations using the multiplicative inverse. Let students use tiles to solve similar problems generated by the teacher or textbook. For example:



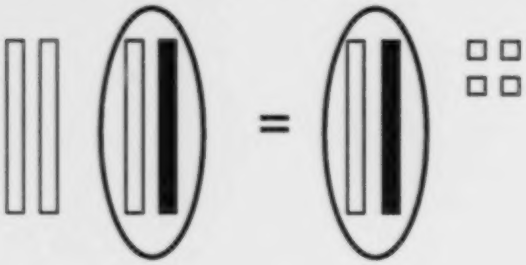
3x + 2 = x + 6

Add **-2** to each side of the equation and simplify.



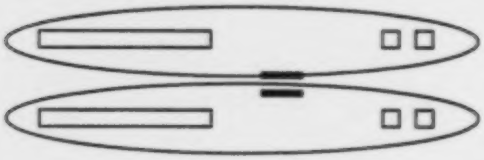
3x = x + 4

Add **-x** to each side of the equation and simplify.



2x = 4

Take half of each side of the equation.



x = 2

B. **Warm Ups (W-26, 28)**

3.3 Solve an equation graphically and by using more than one property of equality.

A. Divide the students into teams. Have the students use any method they wish (including a graphics calculator) to solve the equation. Keep score. Reward appropriately the team with the most correct solutions. Be sure to divide the teams equitably.

B. Solve an equation using paper and pencil. Check using the calculator.

C. Have students use the numbers 2, 3, and 5 (or any three of your choosing) in place of a, b, and c in the linear equation, **ax + b = c**. How many different equations can be written ? Have students record their equations, solve each equation, and compute: the probability that the solution is negative, the probability that the absolute value of the solution is 1, the probability that the solution is a fraction, and the probability that the solution is irrational.

D. **How Do They Fit? (A-11)**
Students will assemble a 3 • 3 array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter.

E. **Warm Ups (W-18, 26, 32)**

3.4 Solve an equation which contains similar terms.

A. Use algebra tiles to combine like terms and solve equations.

3.5 Solve an equation which has the variable in both members.

A. Given a set of problems with variables in both members, ask the students to solve for "Y". Use a calculator to find a solution to each problem.

B. **Equation Relays**

- Divide the students into teams of three. Number each student in each team.
- Distribute the activity sheets (**B-113, 114**) to each team.
- Student #1 should write the first step for solving the first equation and then pass the sheet to student #2. Student #2 should write the next step for solving the equation. The sheet should continue to be passed until the equation is solved.
- For the next equation have student #2 start the process, and so on.
- Continue until all ten equations are solved.

3.6 Solve an equation in which the numerical coefficient is a fraction.

A. Use the relay format described in **Equation Relays (3.5B)** to solve equations with fractional coefficients.

3.7 Solve a formula for one of its variables or find the value of a variable when values of the other variables are given.

A. **Warm Ups (W-54, 60)**

3.8 Use problem solving skills to solve real world and "word" problems which involve a linear equation or a formula.

A. Ask the students to bring in a set of data from the newspaper or a magazine. In groups, develop problems from the data. Find the solutions using any appropriate method. Have pairs of students report on their findings. See **B-16**.

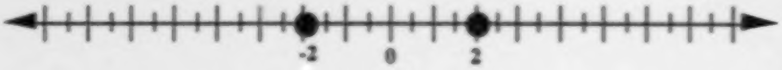
3.9 Solve a simple equation involving absolute value.

A. Ask the students to work in pairs and name themselves A and B. Ask the "A"s to solve **$-x + 2 = 5$** and ask the "B"s to solve **$x + 2 = 5$**. Have the students compare their solutions and substitute the solutions in **$|x| + 2 = 5$**. Ask students to comment.

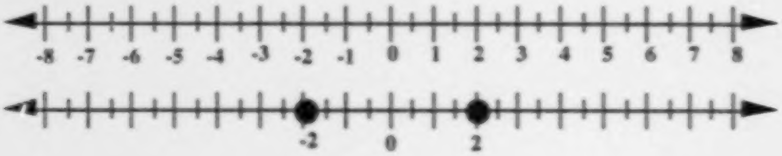
B. Select an equation using absolute value. Take **$4 + |x| = 7$** as an example. On the calculator let **$Y1 = 4 + |x|$** and let **$Y2 = 7$**. Graph and identify the intersection(s) and record the results.

C. Use transparencies of the number line **(B-19)** to locate the solutions to simple absolute value equations. For example:

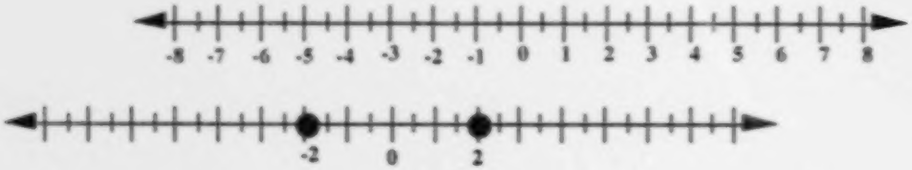
Solve the equation **$|x + 3| = 2$**.
Since the absolute value is 2, mark one of the blank number lines in this manner.



Overlay the newly marked line with one of the numbered lines.



Notice the expression inside the absolute value symbols, **$x + 3$**. In order to find the value of **x** we add **-3**. Visually that means we move the new line three units to the left.



The new "locations" for -2 and 2 now identify the solutions for **x**, -5 or -1.

Using equations from their textbook, let several students solve equations with transparencies at the overhead. Provide copies of **B-19** to the students so that, working alone or in pairs, they can separate the lines and solve absolute value equations as part of classwork and/or homework assignments.

D. **Calculator Tips (W-9)**

E. **Warm Ups (W-36)**

3.10 Solve a simple equation containing a radical.

3.11 Find the solution set for a linear inequality when replacement values are given for the variables.

A. Play **Take a Bow**. Give students cards with a "place" on the number line. Ask them to order themselves like a number line. Then direct the students to "take a bow" if their "place" is greater than 3, then less than 1/2, or greater than or equal to -5. Repeat with another group of students.

B. **Warm Ups (W-24)**

3.12 Solve a linear inequality by using transformations.

A. Use a relay format to practice solving inequalities.

B. Select an inequality (example: **x - 5 < 2**). Enter each side of the inequality on the calculator (**Y1 = x - 5** and **Y2 = 2**). Graph, locate the intersection, and identify for which values of x is the left side above (greater) or below (less) the right side. Solve by hand and compare results with the calculator's results.

C. **How Do They Fit? (A-11)**
Students will assemble a 3 • 3 array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well-constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter.

D. **Warm Ups (W-20)**

3.13 Use inequalities to solve problems.

A. Look for interesting contexts in which inequalities can be used to describe the situation. Here is an example.
In football, the place kicker can score points two different ways. He can score three points with a field goal or one point with a point-after-touchdown (PAT). The coach expects his kicker to score at least 50 points during the season. How many field goals and PATs could the kicker score?

B. **Challenges (W-54)**

3.14 Find the solution set of combined inequalities.

A. Select an inequality (example: **$-2 < x + 7 < 6$**). Enter each part of the inequality on the calculator (**$Y1 = -2$**, **$Y2 = x + 7$**, and **$Y3 = 6$**). Graph, locate the intersections, and identify for which values of x is the center part between the other two parts. Solve by hand and compare results with the calculator results.

Essentials for Instruction	Use samples of finished product as models. Clarify criteria and format when giving written assignments. Use an uncluttered, consistent format for worksheets. Use concise directions, written and oral. Provide time frame for long-range assignments.

The learner will demonstrate an elementary understanding of relations and functions.

4

4.1 Graph and locate sets of real numbers on the number line.

A. Create a number line on a blacktop, gym floor, foyer, or any large area. Have the students locate a point or points by walking to the point(s), starting at zero.

B. Create a number line on the chalkboard or bulletin board. Give each student a card with a number on it. Have each student (or group of students) place his numbers on the line in the appropriate place.

C. **Connections to the World (W-3)**

4.2 Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.

A. Create a coordinate plane in the classroom, on the blacktop, or some large area. Have the students do the "Algebra Walk" by walking to the points you ask them to find. Ask the students to work in pairs. Give each pair an index card with an ordered pair to locate on the coordinate plane. Ask one to be a walker and the other a checker.

B. Create a coordinate plane on a shower curtain liner using markers or gym tape. Place velcro dots at the intersections. Give the students points to locate. Place a dot at that point. (Note: A shower curtain liner is inexpensive and can be used repeatedly, especially if gym tape is used.)

C. **Teacher to Teacher (W-27)**

D. **Connections to the World (W-37, 59)**

4.3 Find the distance between two points on a number line.

A. Create a number line in the classroom or some other large area. Do the "Algebra Walk" by asking two students to walk to the endpoints given and determine the distance by walking the line.

B. Ask students to draw a number line for the information comparing heights of mountain peaks in the United States.
(1) Mt. Mitchell, the tallest peak in North Carolina is 6,684 feet tall.
(2) Mt. McKinley, Alaska, the tallest peak in North America, is 20,320 feet tall.
(3) The elevation of Pike's Peak, Colorado is 6,210 feet less than Mt. McKinley.
(4) The elevation of Grandfather Mountain, NC is 720 feet less than Mt. Mitchell.
(5) How much taller is Mt. McKinley than Grandfather Mountain?
(6) How much taller is Pike's Peak than Mt. Mitchell?
(7) If Death Valley, CA is 282 feet below sea level, how much taller is Mt. McKinley?
(8) Have each student look up elevations of at least one more mountain and add to the number line.
(9) Make a bulletin board to display class results.

4.4 Graph a relation on the coordinate plane.

A. Create a graph on the coordinate plane described in **4.2A**. Give each student a different number for **x**. Have the students begin by walking a line that represents ordered pairs satisfying $y = x$, then everyone add 2 to **x**. Go back to the original line and take the absolute value of **x**. Continue using other modifications of **x**.

B. **Highs and Lows**

Students need to make a chart in their notebook to record the daily high and low temperatures for a one month period. Since there are two quantities (time and temperature) with which they will be dealing, students need to discuss and reach an agreement on which quantity is dependent and which is independent. At the end of the month, students need to create three graphs (daily high temperatures, daily low temperatures, and a composite of the two). Students will find a best-fit line (manually and using the calculator) and make predictions for the next month based on their equations. Students should calculate the average high and low temperatures for the month and compare with their graphs. Students should use the calculators to explore the other best-fit possibilities available. (The actual best-fit function would be trigonometric, sine or cosine. This would be more obvious for data collected over the course of a year.)

C. **Connections to the World (W-25, 31, 69)**

D. **Teacher to Teacher (W-51)**

4.5 Distinguish between a relation and a function.

A. Use the coordinate plane activity from **4.2A** and ask the students to be the "points" on a variety of lines. Have them determine if the lines are functions.

B. Use a "These Are - These Aren't" board (See **2.6A** for an example of a "These Are - These Aren't" board). Give the students a picture, description, or equation and have them determine if it is a function. Ask the students to work in pairs.

C. **"Floordinate" Plane**

◆ Create two sets of cards, x-coordinates and y-coordinates, each numbered from -10 to 10.

◆ On the floor, use masking tape to lay out the coordinate axes.

◆ Give each student a card from each set and have them stand at the location indicated by their cards.

◆ Students exchange x- and y-coordinates and locate the ordered pair on the coordinate plane diagramed on the floor.

◆ Identify the new set of locations as a relation.

◆ Investigate to see if the relation is a function. Have all the students face the x-axis. If none of the students have another student standing in front of them, then they are a function.

◆ Repeat the process.

D. **Warm Ups (W-14)**

E. **The Train Analogy**
Think of a function in terms of the following analogy. Consider the domain to be a set of people who ride the train to work each day. Additionally think of the range as the set of station stops along the train line. The "function" of the train is to deliver the passengers to their respective destinations. It is possible that two or more people will get off at the same station; however, it is not possible for one person to get off at two different stops. For many students this analogy clarifies the concept that each domain value, **x** (a person), is associated with one and only one value, **y** (a station stop).

F. **Connections to the World (W-27)**

4.6 Graph a relation given an equation and a domain.

A. Use the coordinate plane from **4.2A** to graph an equation. Ask the students to work in groups. Give each group a different equation.

4.7 Sketch a reasonable graph for a given relationship.

A. Pair off students. Have one student describe a graph in as accurate a manner as possible and have the other student draw the graph. No peeking allowed until the drawing is finished.

B. Give students a situation and ask them to sketch a reasonable graph by just drawing the basic shape. Have students explain their graphs. Here are examples:

The amount of money you earn on a part-time job and the number of hours you worked.
The number of people absent from school each day of the school year.
An individual's height as he ages.
The height of a baseball as it is hit in the air.
The amount of daylight each day of the year.
The distance a car travels at a constant rate over a 3 hour time period.
The amount of money in a savings account over an extended period of time.

C. Have students sketch a graph for the following situation:
A mail carrier on a rural route must slow down as she approaches a mailbox, stop briefly to place the mail in the box, then continue to the next stop. Sketch a graph of the time versus distance as she delivers mail to three customers, has no mail for the next three deliveries and then leaves mail for two more persons.

D. Have students sketch graphs on the same axis to represent the following three students. Let **x** = time and **y** = distance.
Martin walks to school each morning leaving at 8:00 and arriving at school at 8:30. One morning, he walked for 8.5 minutes then returned home as to pick up his homework. He continued back to school.
Juanita (Martin's sister) drives her car to the same school.
Terry (Martin's brother) usually leaves fifteen minutes later but must run to school and arrives about 8:35

E. **Calculator Tips (W-29, 35)**

F. **Connections to the World (W-43)**

4.8 Interpret a graph in a real world setting.

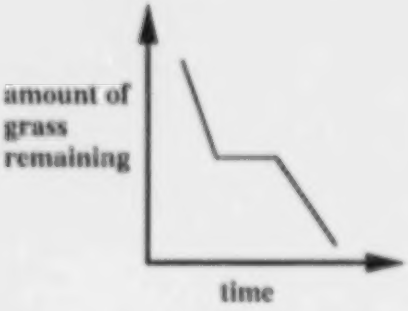
A. Collect a variety of graphs from newspapers, magazines, and other sources. Give a graph to each student. Ask them to write a newspaper article interpreting the graph.

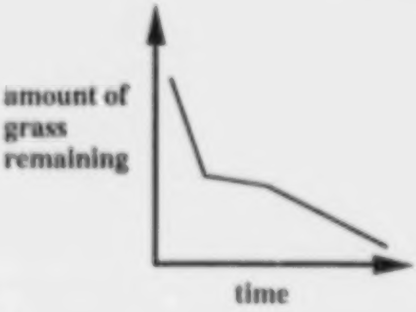
B. **Gulliver's Clothes (A-9)**
This activity will have students working in small groups collecting and recording the circumference of each group member's thumb, wrist, and neck. The data will be plotted on a scatterplot and compared to a linear function. Students will use the scatterplot and linear function to make predictions.

C. Use the computer software, such as ***Interpreting Graphs*** (Wings of Learning).

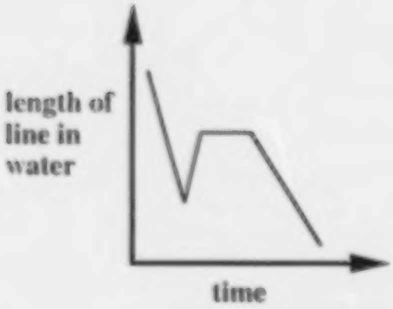
D. Give students the following graphs and have them write a story for each.

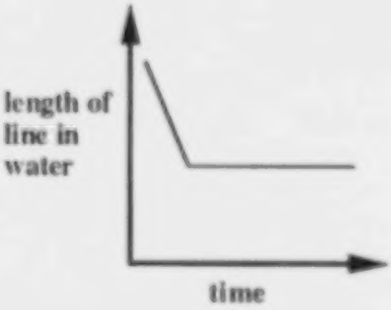
Raoul is cutting the grass:





Sara is fishing:





E. Give students graphs generated from spreadsheets (or have them generate the graphs). In pairs or groups, ask students to write 4-5 questions to be answered from their graphs. Have students exchange papers and answer the questions.

F. **Connections to the World (W-21, 29, 31, 47, 49, 51, 59, 67, 69)**

G. **Warm Ups (W-26)**

4.9 Use a computer or graphing calculator to explore the graphs of functions.

A. After the appropriate research at the grocery store, have students graph on the calculator an equation to represent the cost of a particular food item at d dollars a pound or per item. ($y = d \cdot x$) Students can use the integer screen and trace coordinates to determine the **x** and **y** values. Discuss why this relationship is a function.

4.10 Compare ordered pairs to the line y = x and interpret the results.

A. Create a coordinate plane and demonstrate the line **y = x**. Give each student a card with an ordered pair written on it. In small group or with partners, ask the students to determine where their points lie and its relation to the line **y = x**.

B. Give each student a list of 15 top television shows, movies, CDs, etc. Ask each student to rank the list with 1 being the one most liked and 15 being the least liked. With a partner ask the students to graph their rankings as ordered pairs. *Home Improvement* was a television show that consistently finished in the top 15 during the 1995-96 season. Suppose Student A rates the show a 3 and Student B rates it a 5. Then Home Improvement would be graphed as the ordered pair (3,5). What would the **y = x** line mean in this context? What does a widely scattered set of points mean? What does a set of points close to **y = x** mean? Ask each pair of students to describe and interpret their results.

C. **What Shape Are You? (A-17)**
Determine how many "perfectly square people" you have in your class. Ask the students to measure their height and arm span. Compare the ordered pairs to the line **y = x**.

D. **Warm Ups (W-24)**

Use a multisensory presentation.

Give oral and written directions together.

Emphasize where directions are located.

List steps (oral and written) necessary to complete assignment.

Stress accuracy.

Essentials

for

Instruction

5

The learner will graph and use linear equations and inequalities.

5.1 Determine if data are behaving in a linear fashion.

A. **Teacher to Teacher (W-51)** identifies several sources for useful data. See **B-49** and **B-50** for some examples. Give students opportunities to research and select data, graph it, and determine if it behaves in a linear fashion. Determine lines of best fit and make predictions within the context of the data. See **Scoring and Winning (A-7), The Wave (A-13), and How Do You Measure Up? (A-15)** for examples. Compile the data sets and with modifications use for homework, starter problems, quiz items, and test items.

B. In groups, have students draw two graphs of data that represent linear relationships and two that are not linear. Groups must explain graphs to the class.

C. Give students tables of values that may or may not be linear. Have them investigate which are linear by graphing the data. Ask if they can find a method to determine whether the data behaves in a linear fashion by just observing the tables.

D. This activity can be done in a computer lab with tables of data already on a saved spreadsheet. Have students open the spreadsheet, graph the tables, and record which produce linear graphs. Have students determine what type of table will always be a linear graph.

E. Show pictures of graphs on the overhead. Ask which are linear.

F. Go back to the ***Introduction*** of ***Resources for Algebra***. What is the algebraic expression for the pattern of stars that appear in that section? Create two or three other patterns and share with students. Then have students create their own patterns, collect, edit, and reproduce for students to explore.

G. **Patterns in Perimeter (B-51, 52, 53, 54)**
Students will use perimeter to generate data, find the algebraic expression for the linear pattern, and graph the data that is generated. Although it is expected that students will work individually or in pairs on the pattern sheets, overhead versions of two of the problems are provided for whole class discussion. Using pattern blocks to build the figures is particularly helpful for many students' understanding.

H. **Warm Ups (W-40)**

I. **Connections to the World (W-49, 55, 69)**

5.2 Find the solution set of open sentences in two variables when given replacement sets for the variables.

5.3 Graph a linear equation in two variables.

A. Have students graph and explain applications of linear equations. Here is an example. Tickets to the Carolina Theatre are $6 for adults and $3 for children. If the theatre would like to have a sales income of $600, the equation to model possible solutions is **$6x + 3y = 600$**. Graph the equation. Choose three ordered pairs that are solutions and explain what each represents in terms of the number of adult and children tickets sold.

B. Have students investigate what happens if you change the scales of the axes when graphing. For instance, on graph paper draw graphs of **$10x + 5y = 15$** using different scaling methods. (a) Scale both axes by one, (b) scale both axes by five, (c) scale the x-axis by one and the y-axis by five, and (d) scale the x-axis by five and the y-axis by one. Compare your results.

C. Students can use graphing calculators to investigate families of linear equations. See **The Picture Tells the (Linear) Story (B-55, 56, 57).**

D. Write sets of equations that you think will produce parallel lines.
Write a set of equations that intersect the y-axis at (0,5).

E. Have students enter a table of data from a linear application on a spreadsheet and graph the data. Students can title and label the axes. Have students share printouts with the class and explain the graphs.

5.4 Graph a line given its slope and y-intercept.

A. Discuss how to graph a line using the slope and y-intercept through an application. Here is an example. A health club charges a $75 membership fee and $50 annual dues. Create an equation in slope-intercept form where x is the number of years and y is the total cost. Graph.

5.5 Find the slope of a non-vertical line given the graph of the line, an equation of the line, or two points on the line.

A. Have students work in pairs. Each pair folds a sheet of notebook paper in half and then tears the sheet into two pieces. Then fold each piece into thirds. Each person draws a graph in the middle frame. Students switch papers. Ask them to write on the left frame the slope and y-intercept of the graph and on the right frame the equation of the line.

B. Students can discover the slope-intercept form using graphing calculators. For instance, have students graph each set of equations. Then ask students to describe how the lines are alike and how they are different.

(1) $y = 2x + 1$, $y = 2x + 5$, $y = 2x - 2$
(2) $y = 3x + 2$, $y = 3x - 1$, $y = 3x - 3$

Ask students which values in the equation appear to represent the slope and which the y-intercept. Then ask students to write an equation with slope 5 and a y-intercept of 1. Students can explain their reasoning.

C. Give students two ordered pairs to graph and have them draw the line determined by these points. Ask them to find the slope of the line geometrically and using the slope formula.

D. Play **Guess My Slope and Y-Intercept (B-58)**, a game using a TI-81/82 program.

E. **Boards and Bands**
Give each student (or group of students) a geoboard and set of bands. Have the students mark the x- and y-axes on the geoboard with an erasable marker. Scales can vary so that each peg can represent one unit, two units, five units, and so on. Locate points on the geoboard and discuss how two points are necessary to identify a line. Selecting two points, find the slope of the line which is described by the points. Determine the equation of the same line. Use the graphics calculator to check and compare results. Select other points and repeat the procedure.

F. **What's in a Letter?**
Describe the segments that comprise a capital letter in terms of their slope. Example: the letter contains a horizontal segment, a segment with a positive slope, and a segment with a negative slope. What is the letter? (A) After the teacher has described several letters in this fashion, let students write and share descriptions of upper and lower case letters with each other. Enlarge and overlay letters on a grid and let students compute the segments' slopes and discuss.

G. **Exploring Perpendiculars**
Draw several segments on graph paper and give each student a copy (**B-59**). Have the students find the slope of each segment and record. Have the students fold the graph paper so that the fold is perpendicular to the original line. (This can be done by matching the endpoints and folding.) Have the students find the slopes of the folds and compare the results with the slopes of the corresponding segments.

H. **Challenges (W-70)**

5.6 Describe the slope in a real-world linear relationship using real-world terms.

A. For the equation from **5.9A**, describe the slope in terms of the problem.

B. Have students ask 10-12 teachers the age and mileage of their cars. Have the class draw a scatter plot and a line of best-fit. Ask students to estimate a slope and interpret the slope in the context of the situation.

C. Give the equation $y = 3\ 5x + 25$, have students create a situation modeled by the equation. For instance, let the equation describe the relationship between the number of martial arts classes (**x**) and their cost (**y**) at the YMCA. What does the y-intercept represent? What does the slope represent?

D. **Connecting Units of Measure (A-21)**
Students will measure several objects in the classroom using both centimeters and inches. Students will plot corresponding pairs of measurements on a graph and interpret the information to determine the relationship between centimeters and inches.

E. **Making Sense of Slope**
The handouts **B-60, 61, 62** contain five problems that are related to graphing and slopes of lines. Each problem contains a graph or chart that the students are to analyze and answer questions. Work problem #1 as a class to give students a good idea of what is expected. It would be appropriate for students to work in groups of three or four to complete the assignment.

F. Give students a table of data. Ask them to calculate the slope and interpret. Example:

Hours Worked	Wages Earned
5	28.75
10	57.50
15	86.25
20	115.00

5.7 Write the slope intercept form of an equation of a line.

A. Give students an equation in standard form to graph on the graphing calculator. Show only the final graph on the overhead. Students can check to see if they have the same graph.

B. Sports are appropriate contexts to create problems involving algebra. Here is an example. During the 1993-94 basketball season (October-April), each team in the National Basketball Association played 82 games. On December 1, the Charlotte Hornets had played 13 games and won 8. By January 1, Charlotte had played 28 games and won 16. Create a linear equation in slope-intercept form that describes this trend. Define the slope of your equation with respect to the quantities being discussed. Explain how you created your equation and how you determined the number of games Charlotte should have won. Charlotte finished with 41 wins. How do your "expected wins" compare with the actual wins? Explain the difference, if any. Similar data for the 1995-96 season is provided (**B-63**) for Charlotte and other selected NBA teams. Which team's early season (December-January) performance was best reflected in their final record?

C. **Toothpick Triangles (A-23)**
On graph paper have students plot the ordered pairs. Discuss the meaning of slope geometrically and numerically from the data.

D. **Where's My Line?**
A coordinate plane with 13 globs (**B-64**), some 3 or 4 in collinear arrangements, is put on a transparency. Divide the class into teams of 2 or 3 students. Have a team select a particular set of globs. Each team writes an equation for the line that fits the globs. Each team's line (a separate transparency) is laid on the overhead by the teacher based on the equation the team writes. A team's score is based on the expression $\mathbf{2^n-1}$, where **n** is the number of globs which are on the line.

5.8 Write the equation of a line given the slope and one point on the line, or two points on the line.

A. Find out what your yearbook publisher charges and set up some algebraic situations like the one that follows. Amy said the yearbook company will charge $6300 if 200 yearbooks are printed and $8900 if 400 yearbooks are printed. Write the equation that would describe this linear relationship.

B. Take advantage of data that appears in a newspaper or magazine. Here is an example. Between 1980 and 1990, the number of cable television subscribers increased about 3.6 million per year. In 1980 there were 17.5 million subscribers. Write a linear equation to estimate the number of cable subscribers. Check to see how well the equation estimates subscribers (**B-49**).

C. Have students identify situations that may be modeled by linear equations if a slope and one point are known or if two points are known.

D. **Rise N' Run**
Students will play a game using graph paper and pick-up sticks. On a sheet of graph paper, two students draw the x- and y-axes, determine a scale for the axes, and number each axis. Place the graph paper on the floor. Begin the game by having the two students drop the sticks on the graph paper. Each student reads at least two points on the line made by their stick. Each student earns points by finding: the slope (1 point), the y-intercept (1 point), the line's equation in slope-intercept form (2 points), the line's equation in standard form, $\mathbf{Ax + By = C}$ (3 points). Points are totaled and then summed with $\mathbf{A + B + C}$ to decide the winner. A demonstration at the overhead should occur prior to letting the students work on their own. Students can use the graphing calculator to compare their results with the location of the sticks.

E. **Warm Ups (W-36)**

5.9 Write the equation of a line which models a set of real data.

A. Have students graph data from an almanac or other source (**Teacher to Teacher, W-51**, identifies several sources for useful data. See **B-49, 50** for some examples). Write an equation to model the relationship.

B. Take advantage of data that appears in a newspaper or magazine. Here is an example. In 1986, the 20th Super Bowl was played in New Orleans. The price of a 30-second television commercial for the game was $550,000. In 1993, the 27th Super Bowl was played in Pasadena and the price of a 30-second commercial had risen to $850,000. Write a linear equation that models the change in the price of television commercials for this sporting event.

C. **Scoring and Winning (A-7)**
Have students gather data from the NFL (or similar data from the NBA, NHL, MLB, or local minor leagues) to create scatter plots and find lines of best fit. Students will discuss the characteristics of those lines and make predictions.

D. **The Wave (A-13)**
In a whole class setting, increasingly larger groups of students will perform the "wave". The students will collect and interpret data, determine a linear function of the time to complete the "wave" dependent upon the number of students participating, and use the linear function to make predictions.

E. See **Patterns in Perimeter, 5.1F.**

F. After trying **Basketball: With the game on the line ... (A-27)**, try **Basketball Extension 1 (B-31)**. The **Basketball** activities generally connect probability and statistics with algebra.

G. **Calculator Tips (W-31, 47)**

H. **Connections to the World (W-63)**

5.10 Use the line which models real data to make predictions.

A. After students have created an equation based on data, have them use it to make predictions. Here is a continuation of the example from **5.9B**. According to the equation what was the price of a 30-second television commercial in 1997? What was it actually? Explain any differences. Do the same for the first Super Bowl in 1967.

B. According to postal rate information, **B-65**, what will be the cost of a stamp in the year 2000? In what year do you predict stamps will cost 50¢?

C. **It's All Downhill From Here (A-19)**
Create a linear model to represent rolling a ball down a ramp. Use the model to predict how far the ball will roll down a ramp of given height.

D. **Connections to the World (W-67)**

5.11 Graph a linear inequality in two variables.

A. Select a linear inequality. (Example: $4x + 2y \le 6$) Solve for **y** in terms of x. ($y \le -2x + 3$) Enter the corresponding linear equation in the calculator and graph. Identify the region above (greater) or below (less) the line as the solution set. Using the appropriate calculator function (**Calculator Tips W-27, 33**), graph and record. Choose several points in the region to check in the original inequality. Have the students try several inequalities.

B. See **PlaceKicker, 3.13A**.

C. **Calculator Tips (W-27, 33)**

Use "hands-on" activities. Provide direction when students have several options. Give time for students to organize.	***Essentials*** ***for*** ***Instruction***

6 The learner will graph and solve systems of linear equations and inequalities.

6.1 Use a graph to find the solution of a pair of linear equations in two variables.

A. Select two linear equations and enter them on the calculator. (Example: **Y1 = x - 4** and **Y2 = -x + 3**) Graph, trace, and identify the intersection. Verify with substitution. Record the graph and coordinates of the intersection. Have students try other pairs of equations and record results. Use a friendly range (see **W-17** or **W-25**) for best results.

B. Competition drives much of our economy. Create equations that represent two competitors and investigate under what circumstances you receive the best price for a service or product. Here is an example (research and use actual data). Have students compare total costs for one year at two different video stores. Video City rents video tapes for $3 per tape while Club Video charges a $15 membership fee and $2 a tape. Have students calculate the costs for renting 5, 10, 15, 20, and 25 tapes from each store. Then students can graph the number of tapes versus the cost for both stores. Have students discuss when the costs are the same at both stores. If you regularly rent two tapes per month, which store would be the better buy?

C. Divide students into pairs. Give each pair two systems to graph. Student A graphs the first system at the same time Student B graphs the second system. Have the pair exchange papers to find the solution to the systems from the graph and check the solutions. Each pair works together if difficulties arise.

D. Divide class into cooperative groups. Give each group three graphs of systems of equations and the tables from a spreadsheet. Have groups write the equations for each system, find the solutions from the graphs, and check the solutions using the equations.

6.2 Graph the solution set of a system of linear inequalities in two variables.

A. Select two linear inequalities and enter the corresponding linear equations on the calculator. Using the appropriate calculator function (**Calculator Tips, W-27, 33**), graph and identify the intersection. Record the graph. Have students try other pairs of inequalities and record results.

B. Survey students and create problems similar to the one that follows. Rebecca makes $3 an hour babysitting and $5 an hour when she works at Wendy's. Her mom does not want her to work more than 20 hours per week. Rebecca would like to earn at least $60 a week. Write a system of inequalities that show the number of hours she could work at each job. Graph the system. Write at least four possible solutions.

C. Use information from local businesses to create problems. Here is an example. The Twin Theater charges $6 for adult tickets and $3 for children 12 or under. The theater has 470 seats. The manager wants to have a nightly income (two shows) of at least $4000. Write a system of inequalities for the number of children and adult tickets that can be sold. Write at least four possible solutions.

D. Have students write a system of inequalities whose solution set is (1) a triangle, (2) a trapezoid, (3) a kite, (4) a hexagon.

E. **Challenges (W-54, 56)**

6.3 Use a computer or graphics calculator to solve systems of linear equations.

A. A business will make money if revenues exceed expenses. Discuss with students the break-even point when revenues equal expenses. Have students consider the following hamburger business. The owner pays $20,000 for the franchise and has expenses of $750 per thousand hamburgers. The price of a hamburger is $2.09. Have students write an equation to represent the cost of the business and the equation to represent the revenue. Graph the equations using a calculator. Ask the students to find the number of hamburgers the business needs to sell to break even. What would be the profit if 100 thousand burgers are sold? Many franchises work this way. Have students talk with local owners and share the information with the class.

B. Use business-type situations that arise at school to create problems. Here is an example. The Silver Ratio Band wants to talk to the school principal concerning a contract to play for the Valentine dance. The group is considering three possible rates: (1) the band will charge $3 per person; (2) the school will pay the band $50 plus $2.50 per person; (3) the band will rent the fellowship hall at a local church for $125 and charge $4 per ticket. Which method would be best for the group to use? Have students write a summary of their findings.

C. Ask students to describe the advantages of each method of solving systems of equations: graphing a system of equations on a calculator and using a spreadsheet to analyze a system of equations.

6.4 Use the substitution method to find the solution of a pair of linear equations in two variables.

A. I'm thinking of two numbers. Their sum is 12. The sum of the first with twice the second is 7. What are the numbers ? (17, -5)
I'm thinking of two numbers. Their difference is 9. The sum of twice the first and three times the second is 63. What are the numbers ? (18, 9)
I'm thinking of two numbers. The first is three times as large as the second. Their sum is 48. What are the numbers ? (36, 12)
Ask students to create their own versions of "I'm thinking of two numbers." Collect those from the students, compile and edit, and redistribute for students to solve.

B. Revisit absolute value equations using the substitution method and graphing calculators. Have students graph on the calculator, $y = |x - 2|$ and $y = 4$. Use the trace to find the ordered pairs for the intersection. Next have students solve the system using substitution. (i.e. $|x - 2| = 4$)

Ask students to explain a method for solving an absolute value equation, $|x - 3| = 8$ using a graphing calculator and solving by hand.

6.5 Use the addition or subtraction method to find the solution of a pair of linear equations in two variables.

A. **Challenges (W-60)**

6.6 Use multiplication with the addition or subtraction method to solve systems of linear equations.

A. Give each row of students one of the following sets of equations to solve using the addition method: (1) **$x + 2y = 3$, $4x + 5y = 6$**; (2) **$2x + 3y = 4$, $5x + 6y = 7$**; (3) **$3x + 4y = 5$, $6x + 7y = 8$**; (4) **$4x + 5y = 6$, $7x + 8y = 9$**; (5) **$5x + 6y = 7$, $8x + 9y = 10$**. Call on one person on each row to give the solution. Ask students if they notice a pattern to the equations. Why would they have the same solutions? If all systems were graphed on the same axis, what would they look like?

B. **Challenges (W-60)**

6.7 Use systems of linear equations to solve problems.

A. **Olympic Swimming (B-66)**
With the Olympics occurring every two years (alternating summer and winter games), there are several events in which both men and women compete. For instance, there are winning results for men and women in the 400 meter free-style swimming since 1924. Working in pairs, students will select (or be assigned) an event. Using their calculators, they would determine best-fit linear equations for each of the men's and women's data. If appropriate, use the equations to determine men's and women's performances for 1940 and 1944. (Why were there no results those years?) Predict the winning results for the next several Olympics. Ask the students to determine, according to their calculations, if the women's performance will ever equal or exceed the men's performance in their event. Research as to whether this is likely to happen. At the 1996 Atlanta Olympics the winning times in the 400 meter free-style events were 227.95 seconds for the men and 247.25 seconds for the women.

B. Have students research the transportation costs for travel between cities. Assume that the costs identified represent a linear trend. Here is an example.

	distance	car	air	rail
Raleigh-Charlotte	150 miles	\$47.25	\$145	\$31
Raleigh-New York	650 miles	\$204.75	\$259	\$110

Determine the distance at which driving a car is less expensive than riding the train. When does it become cheaper to fly rather than drive? For what distance is the train the most expensive mode of travel? Identify some advantages and disadvantages for each mode of transportation.

C. **Challenges (W-10, 40, 46)**

D. **Warm Ups (W-16)**

Essentials for Instruction

Teach cues and other listening skills.

Allow time for explanations before giving assignments.

Provide specific feedback on completed work.

Work from concrete to abstract.

Allow some written assignments to be done as a group project.

Use small groups to discuss main ideas..

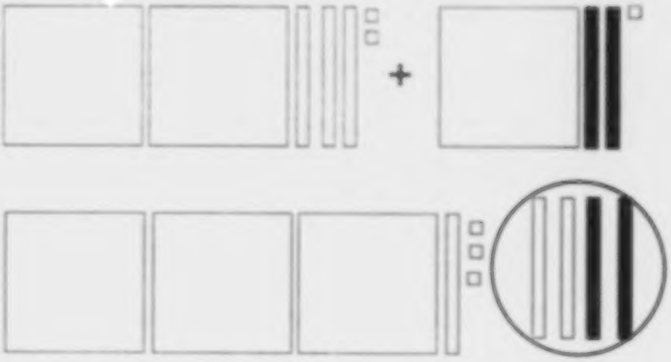
The learner will perform operations with polynomials.

7

7.1 Add and subtract polynomials.

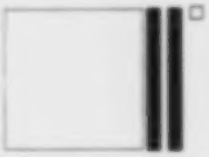
A. Use algebra tiles to demonstrate addition and subtraction of like terms in polynomials.

$(2x^2 + 3x + 2) + (x^2 - 2x + 1)$

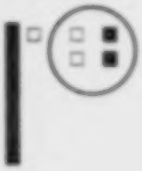


$3x^2 + x + 3$ is the result.

$(x^2 - 2x + 1) - (x^2 - x + 3)$



Take away x^2, $-x$, and add zero (+2 and -2).



Now take away 3.

$(-x - 2)$ is the result.

B. Have students write two monomials whose sum is a monomial and two monomials whose sum is a binomial. Record possible answers. Ask students what generalizations can be made. Continue by having students write two binomials whose sum is (a) a monomial, (b) a binomial, and (c) a trinomial. Again, record possible answers and ask for generalizations. Finally have students write two trinomials whose sum is (a) a monomial, (b) a binomial, and (c) a trinomial. Record answers and discuss generalizations.

C. Look at the following patterns and find the nth terms in each pattern.

I.	II.	III.
$3^2 - 1^2 = 8$	$4^2 - 1^2 = 15$	$5^2 - 1^2 = 24$
$4^2 - 2^2 = 12$	$5^2 - 2^2 = 21$	$6^2 - 2^2 = 32$
$5^2 - 3^2 = 16$	$6^2 - 3^2 = 27$	$7^2 - 3^2 = 40$
•	•	•
•	•	•
•	•	•
nth? ???	???	???

I. $(n + 2)^2 - n^2 = 4n + 4$
II. $(n + 3)^2 - n^2 = 6n + 9$
III. $(n + 4)^2 - n^2 = 8n + 16$

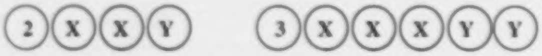
D. After trying **Basketball: With the game on the line ... (A-27)**, try **Basketball Extension 3 (B-35)**. The **Basketball** activities generally connect probability and statistics with algebra.

E. **Warm Ups (W-44)**

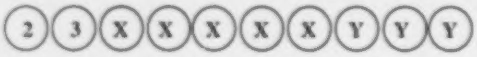
7.2 Multiply monomials.

A. Model multiplying monomials with chips on the overhead. After modeling several examples, have students discuss rules they could use to multiply monomials. Ask students to discuss how this rule differs from adding two monomials.

$2x^2y \cdot 3x^3y^2 =$



$2 \cdot 3 \cdot x^5 \cdot y^3 =$



$= 6x^5y^3$

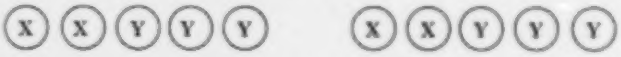
B. Divide students into pairs. Have pairs design a puzzle to match problems with monomial • monomial and solution. Students can cut up puzzle, place in an envelope to exchange with another to solve.

C. Have students write two monomials whose product is $\mathbf{-20x^4y^3}$. Record possible answers on the board/overhead. Have the students make generalizations about possible answers.

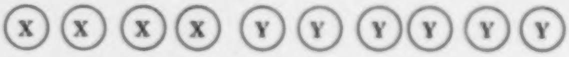
7.3 Find an indicated power of a monomial.

A. Model with chips on the overhead a monomial raised to a power.

$(x^2y^3)^2 =$



$x^4y^6 =$



Have the students come up with a rule for finding a power of a monomial. Also have students discuss how this rule differs from multiplying two monomials.

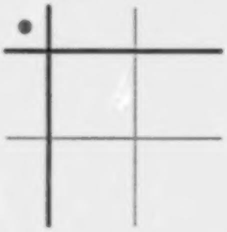
7.4 Multiply a polynomial by a monomial.

A. **I Have ... Who Has ... (A-3)**
Students will listen, perform operations, and respond when appropriate in a round-robin format. Students will need to be able to complete operations with polynomials using paper and pencil. Students will use the format to create their own versions of the activity.

7.5 Find the product of two binomials.

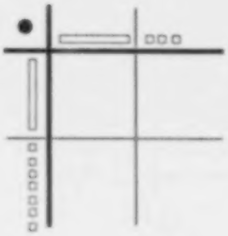
A. **Multiplication With Algebra Tiles (A-31)**
Working in pairs, students will use pairs of binomials as the dimensions of a rectangle. The students will use the algebra tiles to build rectangles of given dimensions and find the area of the rectangle, the product of the binomials.

B. Use the matrix method to multiply binomials (**B-67, 69**). Make appropriate connections with multiplying using algebra tiles.



Fill in the space at the top of each column with the terms from one of the binomials. Fill in the space at the left side of the matrix for each row with the terms from the other binomial. For **(x + 3)(x + 7)**

●	x	3
x		
7		

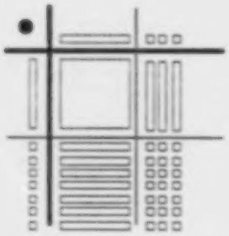


Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

●	x	3
x	x^2	3x
7		

Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

●	x	3
x	x^2	3x
7	7x	21



In the matrix (table) of the products, add along the diagonal from right to left.

●	x	3
x	x^2	3x
7	7x	21

$(x + 3)(x + 7) = x^2 + (3x + 7x) + 21 = x^2 + 10x + 21$.

C. **Polynomial Four in a Row (B-72)**

Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for ***Four in a Row*** is provided (**B-73**) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in Algebra.

D. Before using algebra tiles, demonstrate binomial multiplication using an area model. See **B-74, 75, 76, 77, 78, 79.**

E. **How Do They Fit? (A-11)**
Students will assemble a 3 • 3 array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well-constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter.

F. **Operating With Binomials (A-35)**
Students will fill in the entries for Y_1 and Y_2 with binomials and, using the calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.

G. **Warm Ups (W-46, 58)**

7.6 Multiply two polynomials.

A. Use the matrix (table) method to multiply polynomials. Fill in the space at the top of each column with the terms from one of the polynomials. Fill in the space at the left side of the matrix for each row with the terms from the other polynomial.
For **$(2x^2 + 3x - 4)(3x + 5)$**

●	$2x^2$	$3x$	-4
$3x$			
5			

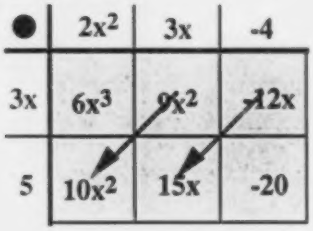
Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

●	$2x^2$	$3x$	-4
$3x$	$6x^3$	$9x^2$	$-12x$
5			

Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

●	$2x^2$	$3x$	-4
$3x$	$6x^3$	$9x^2$	$-12x$
5	$10x^2$	$15x$	-20

In the matrix (table) of the products, add along the diagonals from right to left.



●	$2x^2$	$3x$	-4
$3x$	$6x^3$	$9x^2$	$-12x$
5	$10x^2$	$15x$	-20

$(2x^2 + 3x - 4)(3x + 5) =$

$$= 6x^3 + 9x^2 + 10x^2 - 12x + 15x - 20$$
$$= 6x^3 + 19x^2 + 3x - 20$$

B. **The Month of Algebra (A-29)**
In pairs, students will select several sets of dates in the month of Algebra and complete computations according to teacher directions. By replacing the numbers in a set with appropriate variable expressions and repeating the computations, students will be able to algebraically justify the pattern.

7.7 Divide two monomials.

A. Use the puzzle formats of **How Do They Fit? (A-11), Lining Up Dominoes (A-1), and "I Have ... Who Has ..." (A-3)** to create puzzles for student use. Whenever possible, let students create the puzzles.

B. **Warm Ups (W-50)**

7.8 Divide a polynomial by a monomial.

A. Use the puzzle formats of **How Do They Fit? (A-11), Lining Up Dominoes (A-1), and "I Have ... Who Has ..." (A-3)** to create puzzles for student use. Whenever possible, let students create the puzzles.

7.9 Find a common monomial factor in a polynomial.

A. **Algebra Uno**
Use sets of 100 index cards marked like the layout (**B-80**) of suggested cards (include four wild cards). The game is for three or four players and proceeds much like regular "Uno". In order to lay down a card, the player's card must show a term that has a common factor, other than one, with the card facing up on the discard pile. The player who goes out first wins. A more challenging version could include polynomials on the cards.

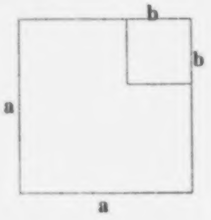
7.10 Factor the difference of two squares.

A. Use the matrix (table) method to factor the difference of two squares (**7.11A, B-68, 69**).

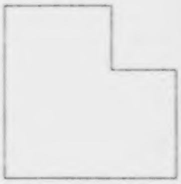
B. Have students reverse the multiplication process to find two binomials that multiply to give these answers. (a) $x^2 - 25$, (b) $a^2 - 16$, (c) $4y^2 - 16$. What pattern do they notice ? Have the students write their rule for finding two binomials that have a product called the difference of two squares. Discuss how factoring is the process of reversing multiplication.

C. Demonstrate $a^2 - b^2 = (a + b)(a - b)$. See **B-81**.

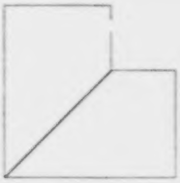
Give directions orally. Cut a square and label the sides **a**. What is the area of this square? Draw a smaller square in a corner of the first square and label its sides **b**. What is the area of this second square?



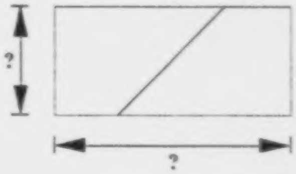
Cut off the smaller square from the corner of the larger square. What is the area of this figure?



Cut the figure as shown. Rearrange these two new pieces so that they form a rectangle.



What is the area of this new rectangle? Have the students discuss the length and width of the rectangle. Ask the students to write an analysis of how this demonstrates the difference of two squares.

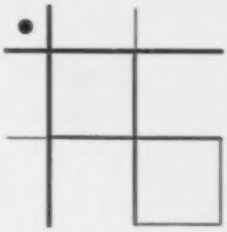


D. **Warm Ups (W-56, 68)**

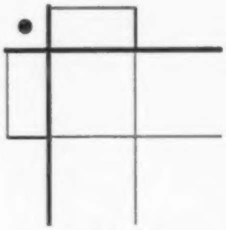
7.11 Factor a simple quadratic trinomial.

A. Use the matrix (table) method for factoring a quadratic trinomial (**B-68, 69**).

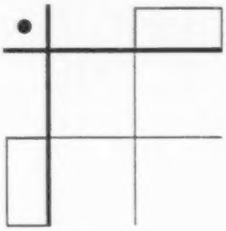
For a quadratic expression of the form **$ax^2 + bx + c$** (**b** can equal zero), place the **ax^2** term in the upper left location (first row, first column) and the **c** term in the lower right (second row, second column).



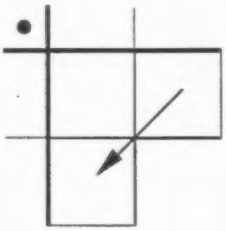
What are two possible factors of ax^2? Place the factors in the locations along the boundary for the first row and the first column.



What are the two possible factors of **c**? Place the factors in the locations along the boundary for the second row and the second column.



Multiply the binomials that are now in the boundary. Do the two diagonal products add up to **bx**? If not, go back and adjust the factors for **c** and/or ax^2.



B. **Polynomial Four in a Row (B-72)**

C. Fill in each square with a digit 0 - 9. You may use each digit only once (**B-82, 83**). Similar puzzles are available commercially.

$$x^2 - x - \square = (x + 2)(x - \square)$$

$$x^2 - 1\square x + 1\square = (x - 2)(x - \square)$$

$$x^2 + \square x + 1\square = (x + \square)(x + 2)$$

$$x^2 - \square x - 24 = (x - 6)(x + \square)$$

D. **Factoring Trinomials with Algebra Tiles (A-33)**
Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

E. **How Do They Fit? (A-11)**

Orally emphasize key words.

Provide clear copy (worksheets, handouts, etc.).

Teach abbreviations germane to the course.

Provide immediate feedback when possible.

Use audiovisuals to introduce and/or summarize.

Have students use logs or personal journals.

Use short answers whenever possible.

Have students repeat directions.

Essentials for Instruction

8

The learner will work with ratios, proportions, and percents.

8.1 Simplify ratios involving algebraic expressions.

A. **Making the Message (B-84)**
Simplify rational expressions to determine a message. Have students use the format to create similar puzzles and after editing, copy to share with the class.

8.2 Solve proportions.

A. **Drive Time (B-85)**
Tina's dad wishes to calculate the number of hours it will take to drive to certain vacation spots. He estimates that he can drive at an average of 58 mph.

B. Use data gathered from various sources to create problems. Here is an example. According to the federal government, it cost the owner/operator of an automobile $47.25 to drive from Raleigh to Charlotte (150 miles). What would be the cost of driving to New York City?

C. Take advantage of your overhead projector. Place a strip of paper 4 cm long on the overhead and have a student measure the length of the image on the screen. Ask students to determine the length of the image of a strip 9 cm long. Relocate the overhead and repeat the exercise.

8.3 Use ratios and proportions to solve problems.

A. After the fact, a detective realizes he forgot to measure the length of a footprint at a crime scene. However, he does have a photograph of the footprint at the scene. Beside the footprint in the picture is a quarter. Explain how to use proportions to estimate the actual length of the footprint.

B. **Estimating Fish Populations (A-25)**
Wildlife officials can approximate the number of fish in a pond to study the fish population and restocking needs. The method that they use is called the capture and recapture method. Students will use a simulation of this method and proportions to calculate a sample fish population.

C. Using tape measures, measure the shadows and height, where possible, of several outside objects. For objects too tall or inconvenient to measure, set up proportions to find the heights of the unknown objects. Compare and discuss results.

D. Create a scale model of the solar system. The focus can be on the size of the sun and planets and/or the distances of the planets from the sun. It may be necessary to use a large field or parking lot on which to set up the "model".

E. For different countries, compare the ratios, population: area. For instance:

	Population	**Area in Square Miles**
China	1,190,431,000	3,696,100
Japan	125,107,000	135,850
Great Britain	58,135,000	94,251
Russia	149,609,000	6,492,800
USA	260,340,990	3,787,319

Follow up with a comparison of those ratios with each nation's gross national product (GNP). Students can look up in an almanac the population and area in square miles for different states in the USA to calculate the population per square mile.

F. Have students calculate the ratio of the speed of the Concorde (2000 miles per hour) to the ratio of the speed of a snail (25 feet per hour). Let students do similar comparisons with other animals and/or vehicles.

G **Basketball: With the game on the line ... (A-27)**
Students, working in pairs, are going to simulate an end-of-game situation 25 times and keep a record of their results. Students will use a random number generator (telephone book, calculator) to simulate shooting free throws. Students will use their experimental results to predict the outcome of the game (win, lose, or overtime). All of the basketball activities (**A-27, B-31, 34, 35**) connect probability and statistics with algebra.

H. **Warm Ups (W-22, 28, 38, 42, 54, 62, 64, 68, 70)**

I. **Connections to the World (W-57, 65)**

J. **Challenges (W-66, 68)**

8.4 Solve problems involving percents.

A. In early April, use the Income Tax Rate Schedules (B-86) to have students calculate taxes.

B. Investigate advertisements that involve percent and use the information to create problems. For instance, a car dealer offers two discounting methods. (1) Take $2000 off of the original price, then 10% more, or (2) take 10% off of the original price, then $2000 more. Which method would give the lower price to the buyer? Explain algebraically which method is best.

C. Collect five or six articles from the newspaper that use percentages. Attach each article inside a file folder and have students write problems related to their articles. Place those in the folder. In cooperative groups, have the students work the file folder problems.

D. Have students in cooperative groups make a list of as many ways as possible to use percentages. Have groups write a problem for each use. Let students report their work to the whole class. Compile the problems and use as homework, starter problems, quiz items, or test items.

E. Have students graph a relationship involving a percentage on a calculator. For example, suppose a music store has all CDs at 15% off. Let x represent the original price and y the discounted price. Graph the relationship. Have students explain how to solve problems using the graph.

F. Percents are introduced as a concept of "part of a whole". For example, a container with 40% orange juice concentrate and the remainder water, is shown as the container with 2/5 orange juice concentrate. In an application situation of doubling, halving, tripling a recipe or any other proportionality, the mathematical thinking becomes "the part juice to the part water". Students can draw a container with two parts juice and reason that the part water would be three. If a container holds twenty ounces, then eight ounces would be orange juice and twelve ounces would be water. Have students come up with four different size containers and determine the part which would be orange juice and the part that would be water. They can develop a table, graph the relationship of the amount of juice to the amount of water. The ratio 2:3 which describes the relationship of the parts can easily be found in the table. The graph can be used to find the amount of water to use for 25 quarts of juice. In addition, the connection between the concepts of percent and ratio of parts can be visualized by the student. Have students investigate the ratio of parts if a solution is 80% alcohol.

G. **Warm Ups (W-14, 44, 46, 48, 52, 56, 58)**

H. **Connections to the World (W-35, 41, 57)**

I. **Challenges (W-48, 52, 58)**

Select materials relevant to students.

Visually illustrate new vocabulary.

Underline or highlight important words in directions or test items.

Reduce copying from the chalkboard or overhead.

Provide a list of all assignments given.

Have frequent review.

Essentials

for

Instruction

9

The learner will explore, graph, and interpret nonlinear equations.

9.1 Graph a quadratic equation.

B. Have students hand graph examples of quadratic equations that are in the textbook. Pair students and ask them to generate a list of characteristics of each equation and quadratics in general.

C. Have students graph $y = 2x - 1$ and $y = x^2 - 1$ on separate graphs. Ask them to write a description of how the graphs are alike and how they are different.

D. **The Picture Tells the (Quadratic) Story (B-87, 88)**
Students can use graphing calculators to investigate families of quadratic equations.

E. Have students use and investigate quadratic equations that could represent physical phenomena. The quadratic equation $y = -4.9x^2 + 30x + 1.5$ describes in meters the height of a baseball that is traveling at 30 meters per second and has been hit from a height of 1.5 meters above the ground. (1) Make a table of values, (2) graph the equation, (3) as time increases, describe the height of the ball, and (4) how does the graph differ from a linear equation?

F. **Calculator Tips (W-53)**

9.2 Use an automatic grapher to find the solution(s) to a quadratic equation.

A. With the graphing calculator, have the students trace and zoom to find solutions for equations set equal to zero. (Example: $x^2 - 3x + 4 = 0$)

B. When given equations like $x^2 - 3x + 4 = 7$, have students enter each side of the equation in the calculator as $Y1 = x^2 - 3x + 4$ and $Y2 = 7$. Graph, trace, and zoom to locate the intersection.

9.3 Solve a quadratic equation when one member is in factored form and the other member is zero.

A. Have students solve equations in factored form with a product of zero from the textbook. Have them explain (1) why does this method work, (2) how can you tell if a solution will be positive or negative, (3) how can you tell if a solution will be non-integral.

B. Have the students reverse the processes. Ask them to write a quadratic equation that has (1) two positive solutions, (2) two negative solutions, (3) two solutions that are fractions, (4) only one solution.

C. Have students discuss why the following is incorrect. Tanya solves a quadratic equation this way.

$$(2x - 1)(x + 2) = 3$$

$$2x - 1 = 3 \quad \text{or} \quad x + 2 = 3$$

$$x = 2 \quad \text{or} \quad x = 1$$

Does $x = 2$ check as a solution to the equation? Does $x = 1$ check as a solution? What is wrong with Tanya's method?

D. **Operating with Binomials (A-35)**
Students will fill in the entries for Y_1 and Y_2 with binomials and, using the calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.

9.4 Solve a second degree equation by factoring.

A. **Factoring Trinomials with Algebra Tiles (A-33)**
Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

B. **Polynomial Four-in-a-Row (B-72)**
Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for ***Four in a Row*** is provided (**B-73**) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in algebra.

C. Use the matrix method for factoring quadratic expressions. See **7.11A**.

D. Have students solve second degree equations by factoring using problems from their textbook. Ask (1) will the method always work and (2) write a second degree equation that cannot be factored.

E. Give students a table of values for $y = x^2 - 2x - 8$.

x	-3	-2	-1	0	1	2	3	4	5
y	7	0	-5	-8	-9	-8	-5	0	7

- From the table, find solutions when $y = -5$ or $x^2 - 2x - 8 = -5$.

- Now use factoring to find the solutions.

- Solve each of the following using the table, then factoring.
  (a) $x^2 - 2x - 8 = 7$
  (b) $x^2 - 2x - 8 = -8$
  (c) $x^2 - 2x - 8 = 0$

- Develop a table with ten ordered pairs using the function $y = x^2 + 2x - 15$. Write four equations you can solve using your table. Solve the four equations by factoring.

9.5 Use an automatic grapher to relate the solutions of quadratic equations and the x-intercepts.

A. Have students relate the solutions of quadratic equations to the graphs of related functions.

(1) Solve by factoring (a) $x^2 + x - 6 = 0$ (b) $x^2 - 8x + 12 = 0$

(2) Now graph on the calculator the related functions and record the graphs. Where does each function cross the x-axis? (a) $y = x^2 + x - 6$ (b) $y = x^2 - 8x + 12$

(3) How can the solutions to quadratic equations be found by graphing the related function? Why?

(4) Find the solutions to the nearest tenth.

(a) $x^2 + 2x - 7 = 0$ (b) $2x^2 - 8x + 3 = 0$ (c) $4x^2 = 8x + 7$

B. **Quadratic Functions (A-39)**

Students will use the quadratic formula to obtain a solution, in radical form, to the related quadratic equation. Using a calculator, they will convert the solutions to decimals, rounding to tenths. Students will then graph the quadratic function on a graphics calculator. They will use the zoom and trace keys to estimate the x-intercepts of the function to the nearest tenth (or they may use the table function).

9.6 Understand that the vertex provides the maximum or minimum value of the function.

A. Have students graph some quadratic equations which model a situation they may be familiar with (**B-89**).

B. **Max-Min**

Give students three points to graph on graph paper. Have them sketch in a parabola that includes the three points and estimate the maximum or minimum value for the parabola. Have the students share their results. After several examples, give students two points with which to work. Again, have students share their results. How many points are needed to determine a parabola?

C. **Open Boxes (A-41)**

In groups, students will collect and analyze data from the construction of several boxes and estimate the maximum volume. Students will use an algebraic model to determine a maximum volume and compare it with the experimental results.

D. **The Maximum Garden (A-43)**
Students will use a table to list possible values for the dimensions and area of a garden space. Students will write an equation to graph the width of the garden versus the area. This will be a quadratic function with a maximum value.

9.7 Solve a quadratic equation in which a perfect square equals a constant.

A. Use the cover-up method to have students solve equations such as $(x + 2)^2 = 49$. On the overhead, write $(x + 2)^2 = 49$ and place a square of paper over the term inside of the parentheses. $(\quad\quad)^2 = 49$. So $(x + 2) = 7$ or $(x + 2) = -7$ which means $x = 5$ or $x = -9$.

B. Use an application. Each side of a square patio was increased in length by 5 feet to give it an area of 150 square feet. What was the original length of the patio? If x is the original length, solve for x when $(x + 5)^2 = 150$.

9.8 Solve a quadratic equation by using the quadratic formula.

A. Find quadratic equations that could represent situations with which students may be familiar. Suppose the equation $y = -1.5x^2 + 3x + 10$ describes the height of a diver above the surface of a pool at any time during a dive. (x is elapsed time in seconds and y is height in meters) (1) How tall is the diving platform? (2) After how many seconds is the diver 5.5 meters above the water? (3) After how many seconds does the diver enter the water? (10 m, 3 seconds, 3.8 seconds)

B. **Quadratic Functions (A-39)**
Students will use the quadratic formula to obtain a solution, in radical form, to the related quadratic equation. Using a calculator, they will convert the solutions to decimals, rounding to tenths. Students will then graph the quadratic function on a graphics calculator. They will use the zoom and trace keys to estimate the x-intercepts of the function to the nearest tenth (or they may use the table function).

9.9 Use quadratic equations to solve problems.

A. **Shuttle Launch (A-45)**
Working in pairs, students will use a pair of quadratic equations to identify the critical points along the flight of the solid rocket boosters (SRB) that are used to launch the space shuttle. Students will connect algebraic ideas (intersection, vertex, x-intercept, and evaluating expressions) with points along the flight path (engine shutdown, maximum altitude, splashdown, and altitude verses elapsed time).

B. Use quadratic equations to represent a commercial situation. The profit of a business can be described by the equation $\mathbf{P = 1.8T^2 - 20T + 250}$, where **P** is the profit in thousands of dollars and **t** is the number of years since 1985 (**t = 0** corresponds to 1985). Describe the profit trend over the last ten years. Use the model to predict in what year the profit will be double that of 1985.

C. **Calculator Tips (W-67, 71)**

D. **Warm Ups (W-72)**

9.10 Determine if a set of data represents an exponential function.

A. **Teacher to Teacher (W-51)** identifies several sources for useful data. See **B-91** for some examples.

B. Begin with a large number of dice (30+). Place the dice in a cup and roll them. Remove all the dice that show 3. Roll the remaining dice and again remove the 3s. Continue the process until there is only one or two dice remaining. Keep a record of the results (**B-92**) for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $\mathbf{y = N \cdot (5/6)^x}$, where **N** is the number of dice with which you begin , **x** is the number of rolls, and y is remaining dice?

With a large number of dice (30+) handy, begin with two dice. Place the dice in a cup and roll them. For every die that shows a 3, add another die. Roll the dice and again add a die for each 3 that appears. Continue the process until all of the dice are used. Keep a record of the results (use **B-92**) for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $y = \mathbf{N \cdot (7/6)^x}$, where **N** is the number of dice you begin with, **x** is the number of rolls, and y is the new dice total?

C. Begin with a large number of coins (30+). Place the coins in a cup, shake, and dump on the desk top or floor. Remove all the coins that show HEADS. Shake and dump the remaining coins and again remove the HEADS. Continue the process until there are no coins remaining. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $\mathbf{Y = N \cdot (1/2)^x}$, where **N** is the number of coins with which you begin, **x** is the number of rolls, and y is remaining coins?

With a large number of coins (30+) handy, begin with two coins. Place the coins in a cup, shake, and dump on the desk top or floor. For every coin that shows HEAD, add another coin. Shake and dump the coins and again add a coin for each HEAD that appears. Continue the process until all of the coins are used. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $\mathbf{Y = N \cdot (3/2)^x}$, where **N** is the number of coins with which you begin, **x** is the number of turns, and y is the new coin total?

D. Have students compare tables of values for a variety of equations (**B-93**).

E. Give students tables of values and ask them which represents an exponential equation (**B-93**).

F. Give students pictures of graphs and ask them which represents exponential data (**B-93**)

G. **Use Your Imagination (B-94)**
Three problems are presented that are mental experiments involving "folding" a single sheet of paper in half many times. These are good illustations of the power of exponential growth. The solutions are the Sears Tower in 23 folds, Mount Everest in 27 folds, and the Moon in 42 folds.

H. **The Dinosaurs Bite the Dust (B-95)**

Simulate the extinction of the dinosaurs using dice to create an exponential model. Point out to students the random nature of the dinosaurs' demise yet we are able to discover a mathematical model for the event. The two theories which try to explain the dinosaurs' disappearence are only provided to the teacher. The teacher can share this information with the students or expect the students to research the topic.

Sixty-five million years ago Earth experienced a global extinction event so severe that it defines the boundary between the Cretaceous (K) and Tertiary (T) geological periods. Causing extinctions on both the lands and in the oceans, that event is referred to as the K-T extinctions. The dinosaurs became extinct during the K-T extinctions.

Of all the theories ever devised for cause of the K-T extinctions, only two remain and they are the focus of intense scientific debate. One, the ***asteroid-impact winter*** theory created by Nobel laureate Luis Alvarez, states that a giant asteroid struck Earth 65 million years ago. It blasted dust into the stratosphere that blocked out sunlight and plunged Earth into a dark, frozen winter.

The other, the ***volcano-greenhouse*** theory originated by Dewey M. McLean, relates the K-T extinctions to a major perturbation of earth's carbon cycle caused by the Deccan Traps Mantle Plume Volcanism in India. This was one of the greatest volcanic events in Earth history and its main eruptions began 65 million years ago. The Deccan Traps released vast quantities of the greenhouse gas, carbon dioxide (CO_2), onto Earth's surface, trapping heat from the sun, and turning Earth's surface into a hot, sterilizing "greenhouse."

It has been estimated that the dinosaurs disappeared in 100-300 years. Using the function $\mathbf{y = 500000 \cdot (G)^x}$, where $\mathbf{x}$ is the number of years since the extinction event, $\mathbf{y}$ is the remaining dinosaurs, and $\mathbf{G}$ $\mathbf{(.01 \le G \le .99)}$ is the rate of population decline, have students determine a rate of decline when there are 100 dinosaurs left after 300 years.

I. Using the North Carolina population information, **B-65**, predict the population in 2000. In what year should the population of North Carolina reach eight million?

9.11 Use formulas, calculators, and automatic graphers to explore and solve problems involving exponentials.

A. **Problems of an Exponential Nature (B-96)**
Many problems like these are available in textbooks. Expect students first to investigate the situation in a table, using their calculators for computation. Then students can discover and discuss patterns in a graph of their data. Students can explore curve-fitting with their calculators and/or discuss how to use the $\mathbf{y = a \cdot b^x}$ form to model the problem.

B. **Patterns with Exponential Equations (A-47)**
Students will graph equations in which the base **b** is a positive number greater than 1. They will investigate what happens as **b** increases and describe the pattern. Next, students will graph equations in which the base **b** is between 0 and 1 and describe this pattern.

C. **Calculator Tips (W-61, 65)**

D. **Connections to the World (W-69, 71)**

E. **Warm Ups (W-70)**

F. **Challenges (W-72)**

Essentials ***for*** ***Instruction***	Vary test format (written, oral, short answer, essay, multiple choice, true-false, matching, computation, yes-no, performance testing). Use a grading system that reflects the varied activities of mathematics instruction. Provide feedback to parents.

Week by Week Essentials (W)

Week by Week Essentials

1

Writing Ideas

What can I expect from you in my class this year?
What do you expect from me as your teacher this year?
Write a math bibliography.
Tell about your triumphs and disasters.
What do you like about learning math?
What do you not like?

Connections to the World

(Engineering) Drag force is a vehicle's resistance to moving through air. Much of an automobile's power is needed to overcome it. Decreasing the surface area of the front of an automobile and recessing bumpers and door handles helps to reduce the drag force. Have students find and explain formulas related to drag. Design an auto of the future that minimizes drag force.

The WORDS are ...

associative

commutative

distributive

Teacher to Teacher

Curriculum Resource Organizer: When trying to correlate the textbook to the Algebra I Standard Course of Study, the format on **B-5 - B-11** may be helpful. Beside each objective the pages in the textbook can be listed and one can quickly identify the parts of the curriculum which are not addressed in the textbook. Additional resources and materials can be listed with the appropriate objective in the third column. Activities from other sections of ***Resources for Algebra*** can be matched with the textbook being used. Working with another teacher (or several other teachers in your district) will provide an opportunity to share ideas and resources.

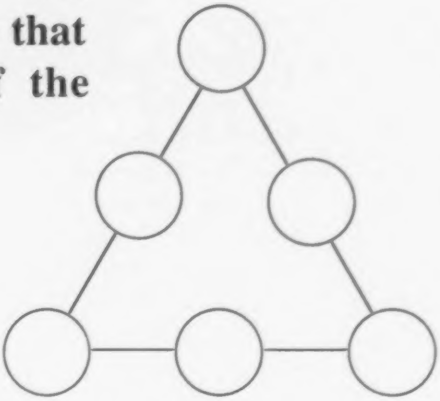
Calculator Tips: Evaluating an Expression

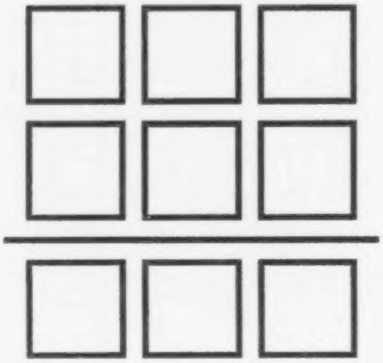
- Open **Y=**; key in the expression for **Y1**; **QUIT**.
- Key the value for the variable; **STO>**; Key the variable; **ENTER**.
- On the **TI-81: 2nd VARS (Y-VARS)**; **1:Y1**; **ENTER.**
- On the **TI-82: 2nd VARS (Y-VARS)**; **1:Function**; **1:Y1**; **ENTER.**

Warm Ups

◆ Place each of the digits 1 - 6 in the circles on the sides of the triangle. Arrange the digits so that the sums along the sides of the triangle differ by

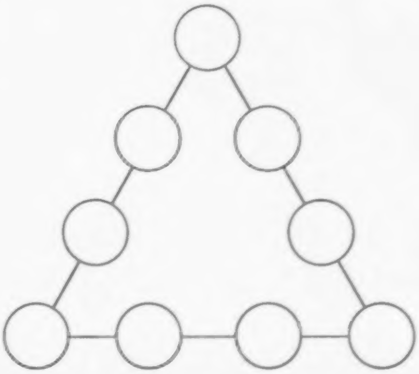
a. 0 (all sums are the same)
b. 1
c. 2
d. 3
e. 4





◆ Place each one of the digits 1-9 in a square to make a true addition problem with the sum very close to 500. Each digit can be used only once.

◆ Place each of the digits 1 - 9 in the circles on the sides of the triangle so that the sum on each side is 23.



Challenge

Find the sum of 1/3 + 1/9 + 1/27 + ...

Writing Ideas

Explain the differences among squaring a number, taking the square root of a number, and multiplying a number by two.

Week by Week Essentials

2

Connections to the World

(Science) Temperature is an important variable to consider when studying natural or man-made phenomena. For instance, snow crystals are shaped differently depending upon the temperature. Graphs are one way to display and discuss snow crystals.

Temperature (°F)	Crystals
below -8	hollow columns
-8 to 3	sector plates
3 to 10	dendrites
10 to 14	sector plates
14 to 21	hollow columns
21 to 25	needles
25 to 32	thin plates

The WORDS are ...

evaluate

expression

Teacher to Teacher

Pacing Guides: No two teachers are alike and no two teachers necessarily teach the topics in Algebra I in the same sequence. In order to assist teachers in the implementation of the revised Algebra I curriculum, several pacing guides were developed for Algebra I and the two year algebra sequence and first shared with the Algebra Pioneer group in 1992. They use a box plot format with the boxes identifying the times at which to give greatest emphasis to particular topics. See **B-1 - 4**.

Calculator Tips: Raising an Expression to a Power

- If an expression has more than one term enclose it with parentheses.
- To raise an expression to the second power: Key the expression; **x^2**; **ENTER.**
- To raise an expression to the third power: Key the expression; **MATH**; **3; ENTER.**
- To raise an expression to any power: Key the expression; **^**; key the power; **ENTER.**

Warm Ups

◆ **Evaluate:** 2(-3)   -9 + 17   -10 + -2
15 • -2 + 17   (33 + 6) ÷ 5

◆ Daniel is the running back for the high school football team. He carried the ball 10 times in the game Friday. Determine his total yardage for the game if his yardage for each carry is: 8, -9, 17, 0, 6, 3, -8, -2, 12, 4.

◆ Let a = 2, b = -3, and c = 5. Find the values of the following:

1. b + c
2. ab - c
3. c ÷ a
4. c - b
5. 3b + c

◆ Complete the magic square by writing each sum in the corresponding square. The sums of the rows, columns, and diagonals are the same.

a. -12 + 12
b. 9 + (-10)
c. 17 + (-13)
d. -12 + 17
e. 21 + (-20)
f. -11 + 8
g. 9 + (-11)
h. -7 + 10
i. 16 + (-14)

a	b	c
d	e	f
g	h	i

Challenges

What is the last digit of 2^{1000}?

What are the last three digits of 7^{9999}?

Week by Week Essentials

3

Writing Ideas

Explain the difference between -5^2 and $(-5)^2$.

Connections to the World

(Consumer) Ask the students to review their parents' telephone bills. Using the data, write a formula for determining the amount you pay. Compare the formulas from the different telephone companies.

The WORDS are ...

formula

algorithm

Teacher to Teacher

Accommodations for Individual Students: Knowing how knowledge is constructed increases the importance of how to use thinking or cognitive abilities. Students and teachers become aware of the power of cognitive strategies such as information gathering, organizing, analyzing, integrating, generating, and evaluating in learning new concepts. Teachers help students to develop strategies which enable students to understand mathematical ideas. For example, information gathering requires a learner to listen, observe, use visual aids, and ask questions. Teachers should take every opportunity to incorporate the teaching of cognitive strategies within the content of a lesson. **B-15** lists ten cognitive strategies identifying behaviors which indicate deficiencies in those and accommodations that encourage growth in developing those strategies. Although the chart and list of accommodations on **B-15** are usually identified with learning disabled students, classroom teachers find that these are appropriate for many students that are not identified LD. The ***Essentials for Instruction*** which appear throughout the Goals/ Objectives pages are accommodations that are important for all students.

Calculator Tips: Scientific Notation

When working with numbers expressed in scientific notation, there are several ways in which one can enter those numbers. Suppose the number to be used is $4.63 \cdot 10^7$.

- On the **TI-81:** Key 4.63 • 10 ^ 7 OR 4.63 • **2nd LOG (10^x)** 7 OR 4.63 **EE** 7.
- On the **TI-82:** Key 4.63 • 10 ^ 7 OR 4.63 • **2nd LOG (10^x)** 7 OR 4.63 **2nd ,** (EE) 7.

Warm Ups

- Find six consecutive positive integers that add to 87.

- With a calculator, find three consecutive numbers that have a product of 97,290.

- Find the units digit in the expression 1997^{1997}. Use a calculator.

- Lauren was trimming a tree next door. She first moved to the middle rung of her ladder to work, then moved up five rungs, then down seven rungs, and finally back up eleven rungs. At that point she was three rungs from the top of her ladder. How many rungs were in the ladder Lauren used?

- If $30 \cdot 15 \cdot 9 = r^2 \cdot 3^2$, then r^2 equals what number?

Challenges

What digits do the letters represent in these two problems? In each problem, each letter can be replaced by only one digit and no digit is used for two different letters.

$$\begin{array}{r} \text{ADD} \\ \text{ADD} \\ \underline{\text{AND}} \\ \text{NBA} \end{array} \qquad\qquad \begin{array}{r} \text{ABCDE} \\ \underline{\times\quad 4} \\ \text{EDCBA} \end{array}$$

Week by Week Essentials

4

Writing Ideas

Explain why it is important to be able to translate an English expression or sentence into an algebraic expression or equation and vice versa.

Connections to the World

(Banking) Banks use machines to sort and count coins. However, when they hand over coins to the Federal Reserve it must be sorted in 50-pound bags. Investigate amounts of money and storage necessary for different sets of coins.

The WORDS are ...

exponents

scientific notation

Teacher to Teacher

Maximizing Class Time: Some teachers have found it useful to start the class with two or three problems to be done as the students come into class. The problems may be a review of yesterday's lesson or from a lesson six weeks ago. As a student finishes give the student a card with the answers. Pass the answer card around the room as the students finish. Get those who finish early to help those who are having difficulty. For a wider range of mathematics, the calendar section of each issue of NCTM's ***Mathematics Teacher*** is an excellent source of starter and challenge problems. See ***Warm Ups*** and ***Challenges***.

Calculator Tips: **Trouble Shooting**

- Make sure symbols, commands, and keystrokes are selected and sequenced correctly.
- If no operation symbol is used, the calculator will multiply.
- When in doubt, use parentheses.
- If nothing happens, **ENTER** again.
- If you have programs in the calculator, do not **RESET** unless you can do without them.
- It never hurts to start over.
- If nothing else works, use the Guidebook that came with your calculator.

Warm Ups

- Give the net charge of the following ions if the number of protons and electrons are given.

	Protons (+)	Electrons (-)
calcium	20	18
aluminum	13	10
bromine	35	36
chlorine	17	18
copper	29	28
iron	26	24
oxygen	8	10
silver	47	46

- The receipts from a school dance include $3.55 in quarters and nickels. How many were there of each coin?

- Which is larger, 3^{40} or 4^{30}?

- What is half of 2^{78}?

- How can (3967 + 3967) • 50 be solved mentally?

Challenges

Find the following sum. $9 + 18 + 27 + ... + 6642$.

If $1 + 2 + 3 + 4 + ... + B = 820$, what is B?

Week by Week Essentials

5

Writing Ideas

To study for an algebra test I usually ...

Connections to the World

(Consumers) Using kilowatt prices from your local power company, determine the cost of operating various household appliances (include televisions, stereos, dish washers, refrigerators, hot water heaters, etc.). Can certain appliances be adjusted to consume less electric power? What are the long range effects?

The WORDS are ...

variable

substitution

Teacher to Teacher

Choosing Appropriate Assessments: A list of assessment methods would include: observation, performance tests, interviews, portfolios, oral questioning, written tasks and tests, class presentations, extended problem solving projects, take-home tests, homework, group work, standardized achievement tests, criterion-referenced tests

- Match the assessment with your learning goals for your students.
- Don't try to do everything at once. Start small and with something you feel you can manage, and build on that as you feel comfortable.
- Don't try to do it all alone. Work with a colleague with whom you can talk to and share ideas. It is extremely important to network. These relationships free you and your colleagues to bounce ideas off each other and receive feedback.
- Communicate to students, administrators, other teachers, and parents what you are doing and why you are doing it. The best defense is a good offense. This will save you a great deal of time in the long run.
- Incorporate informal evaluation into the normal class routine. Evaluation should not come from just a day set aside from instruction.

Calculator Tips: Absolute Value

To find the absolute value of an expression, key **2nd x^{-1} (ABS)**; key the expression; **ENTER.** In a similar manner, key in an equation that uses absolute value. An example: on the calculator **|x+6|** is the same as **abs(x+6).**

Warm Ups

♦ Using the formula for the area of a triangle, $A = (1/2)b \cdot h$, find the value of A if $b = 10$ cm and $h = 3$ cm.
Using the formula for volume of a cylinder, $V = \pi \cdot r^2 \cdot h$, find the volume if $r = 3$ cm and $h = 14$ cm.
Using the formula for the perimeter of a rectangle, $P = 2L + 2W$, find the value of L if $W = 7$cm and $P = 44$ cm.

♦ Evaluate each of the following if $A = 3$, $B = -2$, $C = 3$, and $D = 2.1$

1. B^3
2. D^2
3. $|B - A|$
4. $(C^3 - A^2) \div (B \cdot D)$

♦ The areas of the sides of a box are 42, 54, and 63 cm^2. What is the volume of the box?

♦ On a recent 25-item algebra exam, Scotty's score was 75. The teacher gave her four points for each correct answer and took off one point for each wrong answer. How many items did Scotty get correct?

♦ Show which quantity is greater 2^{100} or 3^{75}.

Challenge

Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$, find the value for $x + y$.

Week by Week Essentials

6

Writing Ideas

Explain why it is necessary to establish an agreed upon order of operations. Use examples to illustrate your reasoning.

Connections to the World

(Astronomy) Astronomers have recently discovered new stars and planets around nearby stars. Knowing that the speed of light is $3 \cdot 10^5$ km/sec, discuss distances to different stars in traditional units (kilometers, miles). One example is a new star, Puenta, discovered $8 \cdot 10^{20}$ light years away. Determine the distance from Earth to Puenta. Discuss how long it takes light to reach the planets from the sun.

The WORDS are ...

distance

absolute value

Teacher to Teacher

Grouping: Grouping and cooperative learning are not synonymous. Students can be grouped for an activity and it not be a cooperative learning situation. Cooperative learning has well-defined criteria and parameters. The groups are carefully chosen and may stay together for some time. On the other hand you may wish to group students for quick tasks or assignments. A group of two or three may work together to find a quick solution to a problem. This gives all of the students an opportunity to talk about the problem with someone. In these small groups, a student can have the problem clarified, get feedback about her thoughts, make suggestions for solving the problem, and explain it in her own words. Groups of three, four, or five may have a larger task to do without the structure attached to cooperative learning. These groups are useful and do serve a valid purpose in the mathematics classroom.

Calculator Tips: Generating the Sequence of Powers of a Number

- Raising a number to a power is easy with the ^ key.
- Key the base number; key ^; key the power desired; **ENTER.**
- To see the sequence of powers of 3, try this.
- Key 3; key *; key 3; **ENTER.** ($3^2 = 9$) Key *; Key 3; **ENTER.** ($3^3 = 27$) **ENTER.** ($3^4 = 81$) Continue to **ENTER** to see increasing powers of 3.

Warm Ups

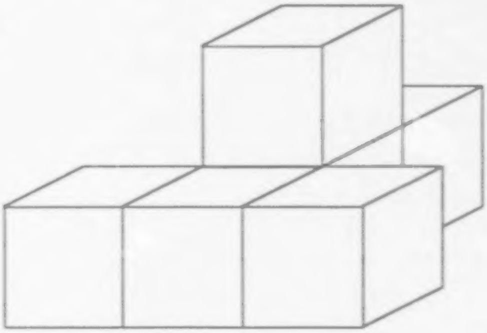
◆ **Write TRUE or FALSE.**

1. If $x = 2$ then $x + 8 \geq 10$
2. If $x = 6$ then $x + 9 < 20$
3. If $y = 5$ then $y - 5 < 0$
4. If $y = 4$ then $4y \neq 16$
5. If $x = 10$ then $x^2 = 100$
6. If $x = 12$ then $3x > 2x + 15$

◆ **Evaluate each expression.**

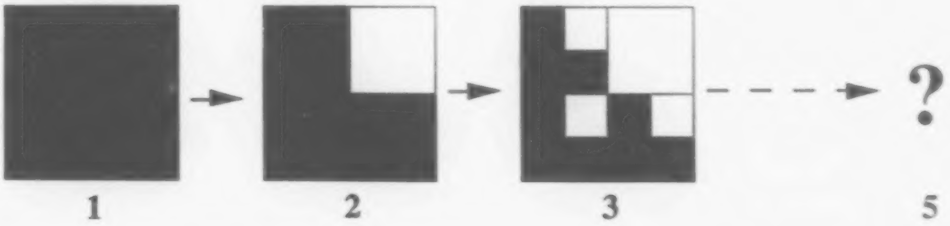
1. $y + 5$ when $y = -7$
2. $x - 4$ when $x = -17$
3. $2y + 7$ when $y = -1$
4. $2t + 5$ when $t = 8$
5. $n - 6$ when $n = 18$
6. $7(p + 2)$ when $p = 8$
7. $-2n - 6$ when $n = 3$
8. $4(q - 5)$ when $q = 10$

◆ **Find the surface area of this figure if each cube has an edge 3.6 cm in length.**



Challenge

In the fifth picture, how much of the original area would remain?



Writing Ideas

Explain how to find the distance between two numbers that have the same sign and between two numbers that have different signs.

Week by Week Essentials

7

Connections to the World

(Traffic Accidents) During the investigation of traffic accidents, evidence that is identified and analyzed are the skid marks left by the cars involved. In the formula **$S = \sqrt{(30df)}$**, **S** is the vehicle's minimum speed in mph, **d** is length of the skid marks in feet, and **f** is the coefficient of friction.

The WORDS are ...

quadrants

origin

coordinate plane

Teacher to Teacher

North Carolina Standard Course of Study (1989), K-8 Algebra: In case you did not know, students entering Algebra 1 should have had many algebraic experiences in the earlier grades. Competency goal 3, patterns, relationships, and pre-algebra, is one-seventh of the K-8 Standard Course of Study. See **B-12** and **B-13**.

Calculator Tips: The Best Window for the TI-81

The window on the **TI-81** is a 96 by 64 pixel grid. Think of numbering the columns of pixels from left to right 1-96 and the rows of pixels from bottom to top 1-64. By choosing values for **Xmin** and **Xmax** to have a difference of 95 (96 - 1) and **Ymin** and **Ymax** to have a difference of 63 (64 - 1), you will have a proportionally correct window. That is, the vertical units will be exactly the same length as the horizontal units. By choosing multiples of 95 and 63 that are derived using the same factor (use a factor of 0.1 to get 9.5 and 6.3), you will have a proportionally correct window.

Warm Ups

◆ **Write YES or NO to indicate whether each relation is a function.**

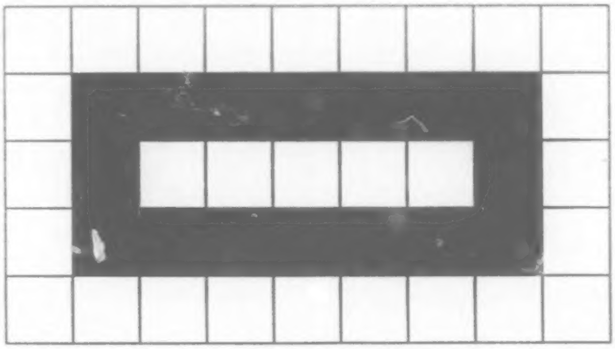
1. { (0,1), (0,2), (0,3) }
2. { (0,0), (1,0), (2,0) }
3. { (1,4), (4,1), (1,5) }
4. { (1,1), (2,2), (3,4) }
5. { (1,2), (2,2), (3,2) }
6. { (3,8), (4,9), (5,8) }

◆ **2 + 7 • 3 + 4**

First add, then multiply. Second, simplify the expression multiplying first, then adding. Third, evaluate in order from left to right. Which is correct? Why?

◆ **Clara's bowling scores for two games were 147 and 131. What score does she have to make in her third game to have an average of 150?**

◆ **What is the chance of hitting the shaded region?**



Challenge

Use the digits 1, 9, 9, 7, and any operations to create an expression equal to the values 0 - 25. Some examples:

1 = 1 + (9 - 9) • 7, 3 = (9 + 9) ÷ (7 - 1)

Week by Week Essentials

8

Writing Ideas

Summarize the mistakes that you made on last night's homework (quiz, test, etc.).

Connections to the World

(Number systems) Did you know that a billion in the United States is a different amount than a billion in England? Here is an example. In the United States, we would say the world population is five billion, six hundred seven million for the number 5,607,000,000. In England, this number is read, five thousand six hundred seven million. A billion in England is 10^{12}. Try this one. In the US, Americans receive 8 billion mail order catalogues a year. How would this number be read in England? Investigate other terminology used in modern number systems.

The WORDS are ...

scatterplot

outliers

Teacher to Teacher

Assignment Calendar: Place a desk calendar for each class (or subject) on a section of your bulletin board. Each day put the assignment on the calendar. Students that are absent can check the calendar to find any assignments that were missed. Near the calendar have a folder or tray for each class where students may drop off papers or pick up handouts for missed assignments.

Calculator Tips: Zooming for a Proportionally Correct Coordinate Plane (TI-81)

A proportionally correct coordinate plane for the graphing calculator is one where the units along the x-axis are the same length as the units along the y-axis. This gives the best view of the shape and position of lines and curves.

ZOOM; **8:Integer**; **ENTER.** This gives you the intervals **$-48 \leq x \leq 47$** and **$-32 \leq y \leq 31$ for the RANGE.**

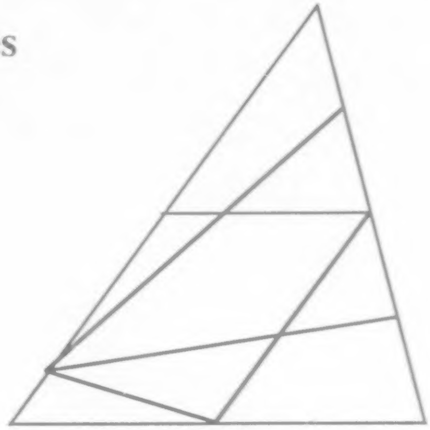
Warm Ups

♦ Choose any number. Multiply the number by four. Add four to the result. Square the original number and add it to the last result. Take the square root of your answer. Subtract the original number from your last result. Why is the final result always two?

♦ John agreed to work a year for $240 and a CD player. At the end of seven months he quit and received $100 and the CD player. What was the value of the CD player?

♦ With a calculator, find two consecutive numbers that have squares differing by 99?

♦ How many triangles are in this figure?



Challenge

How many three digit numbers are there in which the sum of the digits is 24?

Week by Week Essentials

9

Writing Ideas

Find the distance between -7 and -12. Why is it important to use absolute value?

Connections to the World

(Money) Students can investigate currency exchange rates. These were some 1997 rates. Call a local bank to get more current information.

Mexico	1 new peso = $0.13
Canada	1 dollar = $0.74
France	1 franc = $0.18
Japan	1 yen = 8/10 of a cent
England	1 pound = $1.61

The WORDS are ...

equation

open sentence

Teacher to Teacher

Review and Practice: Make review an integral part of your instructional program. Start class with a quick review. Incorporate review within the lesson. Include review exercises in homework assignments. Include review problems on tests. Emphasize to students the value of self-review. Rather than randomly picking review problems, make a plan. Instead of assigning twenty problems from the day's lesson for homework, assign four problems from the five most recently covered sections in the text. Four of these sections will be review while the fifth section will be covering the day's lesson. Remember your students will continue to receive practice of the new skill over the next four assignments. Continue with distributed practice throughout the year.

Calculator Tips: Friendly Range on the TI-81

By choosing values for **Xmin** and **Xmax** to have a difference of 95 (96 - 1), then as you trace along a curve, the **X** values will increase by one unit. Using multiples of 95 will allow you to trace along curves in increments of **X** other than one unit. For instance, if you choose the difference to be 19 (1/5 of 95), then **X** will change in increments of 0.2. If you choose the difference to be 950, then **X** will change in increments of 10.

Warm Ups

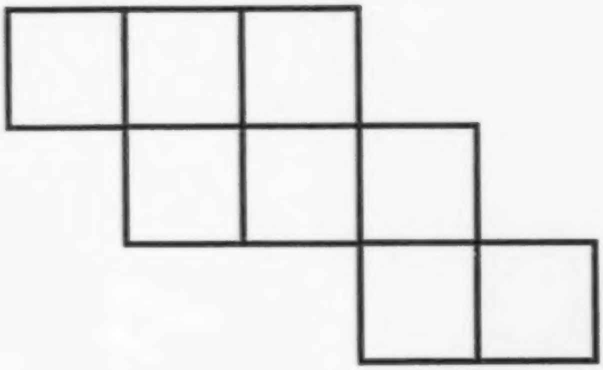
◆ **The formula for the area of a parallelogram is $A = b \cdot h$. What do the variables represent? Solve the formula for h. Find h if the area is 150 square feet and the base is 12.5 feet long.**

◆ **Solve each equation for y when $x = 0$.**

1. $y = 2x + 3$ 3. $y = x - 5$
2. $y = -3x + 4$ 4. $y = (2/3)x - 5$
Name one point on the graph of each equation in 1-4.

◆ **If $x - 4$ is two greater than y, then $x + 5$ is how much greater than y?**

◆ **The area of this figure, made with eight congruent squares, is 2312 cm^2. What is the perimeter of this figure?**



Challenge

Represent the number 43 using seven 7s and any operations.

Week by Week Essentials

10

Writing Ideas

Write a question related to yesterday's class.

Connections to the World

(Computers) In 1978, two California high school students, Laura Nickel and Curt Noll, set the record for the largest prime number found. They used 440 hours on a large computer. This number, $\mathbf{2^{21701}-1}$, has 6,533 decimal digits. A record prime number found in 1985, $\mathbf{2^{216091}-1}$, has 65,050 decimal digits. The computer completed approximately 1.5 trillion calculations in about three hours. Ask students to to find out what the most current record is and provide documentation and why time would be spent on a computer searching for primes or computing π. (See **W-75** for more information.)

The WORDS are ...

inverse

reciprocal

Teacher to Teacher

Homework: One of the biggest difficulties for many teachers is homework. Homework is used to practice concepts or to preview an upcoming topic. How do you collect it, check it, use it, etc., to benefit you and your students?

- Ask students as they come into the classroom to identify a problem from the assignment that needs review or clarification.
- List the numbers of the problems of a homework assignment on the board. As the students come in ask them to put a tally mark beside the problem(s) where they have had difficulty. The teacher can easily glance at the board to see where most of the problems are.
- Put the answers to the assignment on the board or overhead projector. As the students check their work, monitor to see if students have difficulty with specific problems.
- Have a student moderator who reviews the assignment while you walk around and monitor. Be sure to model the role of moderator and give the chosen student a day's notice to prepare.
- Give homework pop tests with questions related directly to the homework assignments.
- Give each student answer keys for the week's assignments. Check to see if they have attempted the problems by monitoring one or two specific problems.
- Identify only one or two problems from the assignment to check or grade. Give the students a warm-up problem. While they are working, grade the chosen problems.

Warm Ups

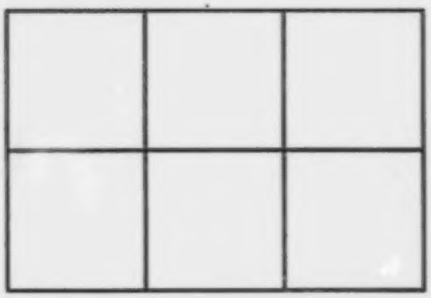
◆ Solve each equation or inequality for x.

1. $2x + 7 = 39$	4. $2x + 7 < 39$
2. $2x + 7 > 39$	5. $7x < 35$
3. $7x < -35$	6. $-7x < -35$

◆ Natalie pays two 40¢ tolls and one 30¢ toll driving to and from work each day. Write a formula for the cost, C, for any number of commutes, N. Use your formula to calculate the total amount Natalie paid for driving to and from work for five days.

◆ Find the smallest four digit number whose square root is an integer. Use a calculator.

◆ How many rectangles appear in this figure?



Challenge

A quaza is a positive integer having at least four different factors and the difference between one pair of factors is the sum of another pair of factors. For instance, 24 is a quaza since $4 + 6 = 12 - 2$. Find the other eight quazas less than 180.

Week by Week Essentials

11

Writing Ideas

Explain the difference between an expression and an equation.

Connections to the World

(Consumers) Ask each student to follow the price of a product he/she uses frequently. Regularly over several months, record the price of the product. Make a graph and discuss the results.

The WORDS are ...

inequality

interval

solution set

Teacher to Teacher

Test Review Bingo: Prepare sixteen review problems for a test. Before class, write each problem on an index card and place into a hat or small box. At the beginning of class, have the students draw a four by four bingo grid on their paper and place the numbers 1-16 randomly in the grids. To review, draw a card, read the question to the class, and have them find the solution. After students have answers, they can discuss their solution methods. Students who have the correct result can cross out the problem's number on their grid. (Students who correctly answer #3 mark off 3 on their grid.) The student who has a row, column or diagonal crossed out first is the winner of the contest. If you have fewer than sixteen problems or time is short, give students "free" numbers.

Calculator Tips: **The Best Window for the TI-82.**

The window on the **TI-82** is a 95 by 63 pixel grid. Think of numbering the columns of pixels from left to right 1-95 and the rows of pixels from bottom to top 1-63. By choosing values for **Xmin** and **Xmax** to have a difference of 94 (95 - 1) and **Ymin** and **Ymax** to have a difference of 62 (63 - 1), you will have a proportionally correct window. That is, the vertical units will be exactly the same length as the horizontal units. By choosing multiples of 94 and 62 that are derived using the same factor (use a factor of 0.1 to get 9.4 and 6.2), you will have a proportionally correct window.

Warm Ups

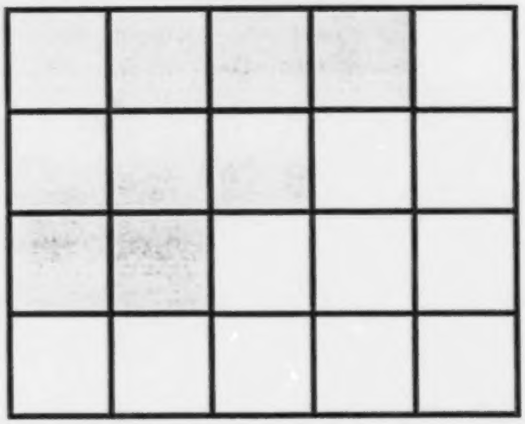
◆ When Alice opened the algebra book, two pages faced her. If the product of the two page numbers is 101,442 what are the two page numbers?

◆ The first five figure skating judges gave David scores of 7.2, 7.3, 7.7, 7.8, and 7.5. What score did he need from the sixth judge to average 7.55?

◆ The product of the ages of a group of teenagers is 11,995,200. Find the number of teenagers in the group and the sum of their ages. Use a calculator.

◆ Write a ratio for each based on the figure.

1. shaded squares to total squares
2. unshaded squares to total squares
3. shaded squares to unshaded squares
4. total squares to unshaded squares



Challenges

What is the units digit of 3^{419}?

Find the remainder when 6^{1997} is divided by 11.

Writing Ideas

Explain the difference between an inequality and an equation.

Week by Week Essentials

12

Connections to the World

(Geography) Using a North Carolina road map, locate places on the map. Survey a grid square to determine the types of towns/cities in the square. How many towns have populations greater than 25,000? How many counties are found in that square? Compare and report the information.

The WORDS are ...

relation

ordered pairs

coordinates

Teacher to Teacher

Grading: Students are very sensitive about grades. You are "unfair" if you do one thing one time and another thing at a different time. They do not forget. Consequently, you are unfair. To reduce this occurrence, be very specific about grading. Determine the criteria before you grade a paper. Let all of your students know what those criteria are. Allow your students to give input about grades when it is appropriate. Select and review a small sample of papers at one time for a picture of the class as a whole. Be selective comments, choose one or two aspects for evaluation or scoring and detailed feedback. Ask students to underline the ideas they really want you to notice. Teach students to assess one another's work.

***Calculator Tips:* Zooming for a Proportionally Correct Coordinate Plane (TI-82)**

A proportionally correct coordinate plane for the graphing calculator is one where the units along the x-axis are the same length as the units along the y-axis. This gives the best view of the shape and position of lines and curves.

- **ZOOM**; **4:ZDecimal.** This will give you the intervals $-4.7 \le x \le 4.7$ and $-3.1 \le y \le 3.1$ for the **WINDOW.**
- **ZOOM**; **8:Zinteger**; **ENTER.** This will give you the intervals $-47 \le x \le 47$ and $-31 \le y \le 31$ for the **WINDOW.**

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Warm Ups

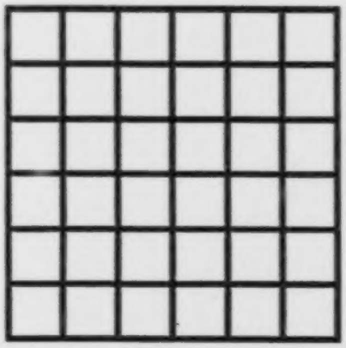
- Solve $2x + 6 \geq 15$ using the values for x from {0, 1, 2, 3, 4, 5}

- Graph the equation $y = x$.
  Complete each ordered pair so that $y > x$.
  (1, __), (-2, __), (-3, __)
  Graph these ordered pairs with $y = x$.
  Where are those points with respect to $y = x$?

- What is the ones digit of 3^{1997}?

- List from greatest to least. 4^{60}, 5^{48}, 3^{72}

- How many squares are contained in a $6 \cdot 6$ square grid?



Challenge

Use three 9s and any operations necessary to create an expression equal to one. Use four 9s. Use five 9s. Describe any patterns you observe. Try again using 5s instead of 9s. Describe any patterns. What about 3s? 6s? Any number?

Week by Week Essentials

13

Writing Ideas

Explain what the graph of an inequality represents. Identify the difference between an open dot and a closed dot.

Connections to the World

(Consumers) Ask students to determine the Top Ten for music singles, discs, books, movies, videos, etc. Follow the list over time to look for trends. Have students select a single entry from one of the categories and follow its climb (and decline) through the charts. Graph.

The WORDS are ...

domain

range

Teacher to Teacher

Individual Chalkboards and Graphboards: Teachers often need for every student to answer a question individually. Individual chalkboards or dry erase boards can be used. Ask the students to bring old socks (without holes) to use as erasers. Drop the chalk or dry pen in the sock for quick availability. To make your own board, laminate card-stock paper. Use overhead pens to write on the boards. Each student will need a damp paper towel with which to erase (baby towelettes work too).

Rolls of graph paper can be purchased from various vendors. To make graphboards, cut the graph paper to fit card stock paper, glue onto the paper, and laminate. Make enough for a class set. Individuals or groups can use overhead projector pens to draw their graphs and share with the class. Give students enough time to clean up so the boards will be ready for the next class.

Calculator Tips: **Friendly Range on the TI-82**

By choosing values for **Xmin** and **Xmax** to have a difference of 94 (95 - 1), then as you trace along a curve, the **X** values will increase by one unit. Using multiples of 94 will allow you to trace along curves in increments of **X** other than one unit. For instance, if you choose the difference to be 47 (1/2 of 94), then **X** will change in increments of 0.5. If you choose the difference to be 940, then **X** will change in increments of 10.

Warm Ups

◆ **Show the results of multiplying both sides of the equation by 15.**

$x/5 + x/3 = 8$

$A/3 + 2 = 1/5$

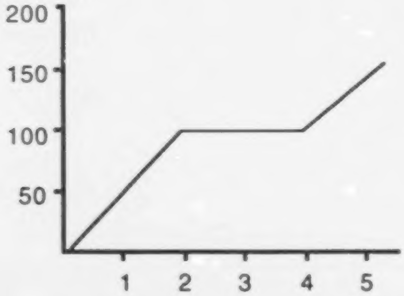
$3y + 2/3 = 4/5$

◆ **Find the mean of -6, 4, 2, 0, -7, 6, -4, -2**

◆ Solve $(3n + 5) \div 2 - 6 = -1$

◆ **Find the sum of the reciprocals of all of the factors of 24; of 60.**

◆ **The graph shows the distance traveled versus the time elapsed. Describe what can be happening according to this graph.**



Challenge

What is the least number of coins necessary to make any amount between 1¢ and 99¢?

Writing Ideas

Why is the solution to an equation not always the answer to the question?

Week by Week Essentials

14

Connections to the World

(Healthful Living) Record personal achievement on fitness tasks periodically over an extended length of time. Discuss personal fitness as a function of time and a dependent relationship.

The WORDS are ...

model

demonstrate

Teacher to Teacher

The Coordinate Classroom: Turn your classroom into a coordinate plane. Each desk should have a location on the plane. Give the students a card with a "location" (ordered pair) on it. Have the students one-at-a-time find their places. You may wish to give students their cards as they walk into the classroom.
Create an outdoor version by painting a coordinate plane on a blacktop. The students can locate their positions using the coordinates written on their cards.

Calculator Tips: **Inequalities on the TI-81**

- To graph **$y < x+ 2$**.
- **Y=**; enter **$x + 2$** for **Y1**.
- Choose **6:Standard** from the **ZOOM** menu. **2nd MODE (QUIT).**
- **2nd PRGM (DRAW)**; **7:Shade**; **-11**; **,**(comma); **2nd VARS (Y-VARS)**; **1:Y1**; close parentheses; **ENTER**.
- To clear the screen: **DRAW**; **1:ClrDraw**.
- Generally, the **Shade** function is defined **Shade(lower boundary, upper boundary)**.
- To graph **$Y \geq x - 4$**, the next two steps are the same as above.
- **2nd PRGM** (DRAW); **7:Shade**; **Y-VARS**; **1:Y1**; **,** (comma); **11**; close parentheses; **ENTER**.
- When graphing with **>**, choose an upper boundary greater than **Ymax**. When graphing with **<**, choose a lower boundary less than **Ymin**.

Warm Ups

◆ Solve each equation.

1. $x + 5 = 12$ 4. $x + 5 = -12$
2. $2x = 18$ 5. $2x = -8$
3. $-5x = 20$ 6. $-5x = -20$

◆ Meredith makes 88% of her free throws. During one game she made 11 out of 15 free throws attempted. Did she shoot as well as usual? What if she made 12 out of 15? 13 out of 15? 14 out of 15?

◆ If a pyramid has ten edges, what is the shape of the base?

◆ Tell whether each statement is TRUE or FALSE. Name the mathematical property each statement illustrates.

1. $2 + 11 = 11 + 2$ 4. $3 \cdot 5 = 5 \cdot 3$
2. $(4 + 1) + 7 = 4 + (1 + 7)$ 5. $7 - 2 = 2 - 7$
3. $(8 \cdot 3) \cdot 2 = 8 \cdot (3 \cdot 2)$ 6. $3 \div 5 = 5 \div 3$
7. $5(10 + 3) = (5 \cdot 10) + (5 \cdot 3)$

Challenge

On a television show, a player receives five points for answering an easy question and eleven points for a hard one. What is the largest integer that cannot be a contestant's total score in the game?

Week by Week Essentials

15

Writing Ideas

Write a description of the graph of an equation. Read the description to another student and see if that student can tell you what the equation is.

Connections to the World

(Newspapers) Ask the students to bring to class graphs found in newspapers over a week's time. Determine the types of graphs that are used most often. Discuss their appropriateness.

The WORDS are ...

justify

rationale

Teacher to Teacher

Grading Journals: Color code student journals by giving students colored stickers to place on the front of their journals. For example, give 6 students blue dots, 6 students red dots, 6 students green dots, 6 students pink dots, and six students purple dots. Use a spinner or colored cube to make selections.

Calculator Tips: Entering and Displaying Data (TI-81)

- Check **MODE** to make sure all settings are appropriate. **2nd CLEAR (QUIT)**.
- Clear all equations in **Y=**; **QUIT**.
- **2nd MATRX (STAT)**; go to **DATA** menu; **2:ClrStat**; **ENTER**; **QUIT.**
- **STAT**; go to **DATA** menu; **ENTER** or **1:Edit**.; enter **x** and **y** values until all data are entered; **QUIT**.
- **RANGE**; enter values for **Xmin, Xmax, Xscl, Ymin, Ymax, Yscl**; leave **Xres = 1**; **QUIT**.
- **STAT**; go to **DRAW** menu; **2:Scatter**; **ENTER**; **QUIT.**

Warm Ups

◆ During the week before Halloween, Nathan ate 140 pieces of candy corn. Each day he ate five more than he ate the previous day. How many candy corn pieces did he eat each day?

◆ With a calculator, find a square whose last two digits are its square root.

◆ Jan scored twenty points, including three free throws. How many three-point field goals did she score?

◆ Complete the magic square by computing the y value of the relation for each x value given below. The rule is each y value is one less than three times each x value.

a. 1
b. 0
c. 5
d. 6
e. 2
f. -2
g. -1
h. 4
i. 3

a	b	c
d	e	f
g	h	i

Challenge

942 digits were used to write the page numbers of a book. How many pages did the book have?

Writing Ideas

Explain why the graph of **y = 2** is a horizontal line and why **x = 3** is a vertical line.

Week by Week Essentials

16

Connections to the World

(Politics) Determine the party membership for North Carolina's congressmen since 1860. Discuss changes and trends and graph. Do the same for US senators.

The WORDS are ...

approximate

estimate

Teacher to Teacher

Graph Transparency Overlays for Data Collection:

- Give out one sheet of graph paper to each cooperative group that already has the x and y axes drawn and scaling marked so that groups have identical graphs.
- Give each group a clear piece of acetate and overhead projector pen. Have students place the clear transparency over the graph paper and place a corner mark where the x and y axes meet. They graph only the data collected for their group.
- Make a transparency of the original graph paper with the same scaling. Overlay each groups' transparency with plotted data over your graph paper transparency to display the data for your entire class.

Calculator Tips: **The Best-Fit Line (TI-81)**

- Enter data.
- **STAT**; **2:LinReg**; **ENTER**. Calculator will display values for **a**, **b**, and **r**.
- The calculator uses the general form of a linear equation, $y = a + bx$.
- The correlation coefficient **r** ($-1 \leq r \leq 1$) provides information about the quality of the linear fit.
- To graph the best-fit equation: **Y=**; **VARS**; go to the **LR** menu; **4:RegEQ**; **GRAPH**.
- To graph the best-fit equation and the scatter plot of the data: **STAT**; go to **DRAW** menu; **2:Scatter**; **ENTER**.

Warm Ups

◆ **Indicate what needs to be done to get x alone on one side of the equation.**

1. $x + 7 = 12$
2. $x - 9 = 15$
3. $x + y = 3$
4. $x - 2y = 7$
5. $-x + 3y = 4$
6. $-2x - 2y = 6$

◆ **If** $p_5 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ **and** $p_4 = 1 \cdot 2 \cdot 3 \cdot 4$, **then simplify** $(p_5 - p_4) \div p_4$.

◆ **Write an equation involving perimeter for each rectangle.**

	length	width	perimeter
1.	x	2	12
2.	x	$x - 2$	20
3.	$2x$	x	18
4.	$x + 4$	x	88

◆ **Simplify:** $(5/6 \div 5/6) + (3/4 \div 3/4) - (1/2 \div 1/2) - (2/3 \div 2/3) + (7/8 \div 7/8) - (3/5 \div 3/5)$

◆ **Which is greater,** 1996^{1997} **or** 1997^{1996}**?**

Challenge

Find three consecutive odd numbers the sum of whose squares consists of four identical digits.

Week by Week Essentials

17

Writing Ideas

Compare and contrast a relation and a function.

Connections to the World

(Consumer) Ask each student to pick a vacation place that they would like to visit. Research the place to determine whether or not you would need to rent a car if you had arrived by air or rail.

The WORDS are ...

solution

verify

Teacher to Teacher

Algebra in the NCTM Curriculum and Evaluation Standards for School Mathematics (1989) : The NCTM Standards articulate algebraic expectations across all grades. In K-4, the Standards include patterns and relationships; in grades 5-8, patterns, functions, and algebra; and in grades 9-12, algebra and functions. See **B-14**.

Calculator Tips: Inequalities on the TI-82

- To graph $y < x + 2$.
- **Y=**; enter $x + 2$ for **Y1**.
- Choose **6:ZStandard** from the **ZOOM** menu; **2nd MODE (QUIT)**.
- **2nd PRGM (DRAW)**; **7:Shade**; **-11**; ,(comma); **2nd VARS (Y-VARS)**; **1:Function**; **1:Y1**; close parentheses; **ENTER**.
- To clear the screen: **DRAW**; **1:ClrDraw**.
- Generally, the **Shade** function is defined **Shade(lower boundary, upper boundary)**.
- To graph $y \geq x - 4$, follow the next two steps from above.
- **2nd PRGM (DRAW)**; **7:Shade**; **Y-VARS; 1:Function**; **1:Y1**; , (comma); **11**; close parentheses; **ENTER**.
- When graphing with **>**, choose an upper boundary greater than **Ymax**. When graphing with **<**, choose a lower boundary less than **Ymin**.

Warm Ups

◆ Show that the numbers from 1 - 25 are either squares or the sums of squares. For example: $10 = 9 + 1$

◆ Show that the numbers from 1 - 25 are the sum of distinct powers of two. For example: $21 = 2^4 + 2^2 + 2^0$.

◆ Write an inequality for each sentence.

1. The difference of three times a number and the number is less than -12.
2. Three more than twice a number is greater than forty-three.
3. If one is added to three times a number, the sum is less than twice the number.
4. If one is subtracted from five times a number, the result is greater than 15.

◆ Brenda bought 15 hamburgers for 79¢ each and 15 orders of fries for 69¢ each. What was the total cost?
Mike had a party for 15 friends. Each friend had a 79¢ hamburger and a 69¢ order of fries. What was the total cost?
What property is illustrated by these two problems?

Challenge

Which pairs of two digit consecutive numbers have squares that differ by a perfect square?

Week by Week Essentials

18

Writing Ideas

Discuss the most difficult homework problem from the last assignment.

Connections to the World

(Stock Market) Pick a company and find the company's stock price. Record the price for several days/weeks. Compute the percentage increase/decrease on a day-to-day or week-to-week basis. Use a spreadsheet to record and analyze the data.

The WORD is ...

function

Teacher to Teacher

Inexpensive Counters: Chips or counters to illustrate integer operations can be made from construction paper and re-enforced with Contact™ paper. Contact paper is the right size to fold over the construction paper to cover both sides. Cut a piece of contact paper equal to the length of a piece of construction paper. The roll width is exactly double the width of the construction paper. Place the construction paper on one side and fold over the contact paper. Cut the re-enforced paper into approximately one inch squares. This works better than laminating since it is stuck to the complete sheet of paper. Two colors of chips can be placed in a small envelope for each group of students.

Calculator Tips: Entering and Displaying Data (TI-82)

- Check **MODE** to make sure all settings are appropriate. .
- **Y=**; clear all equations; **QUIT**.
- **STAT**; **4:ClrList**; key the lists you want cleared, separated by commas; **ENTER**.
- **STAT**; **1:Edit**; enter data (**x** values in **L1** and **y** values in **L2** or other appropriate lists); **QUIT**.
- **WINDOW**; set values for **Xmin, Xmax, Xscl, Ymin, Ymax, Yscl**; **QUIT.**
- **STAT**; go to **CALC** menu; **3:Setup...;** under **2-Var Stats**, select **L1** for **Xlist** and **L2** for **Ylist** (or setup according to other lists used); **QUIT**.
- **2nd Y= (STAT PLOT)**; **1:Plot1**; highlight **On** by **ENTER**ing; highlight the **Type** of graph you want; highlight **L1** on **Xlist** and **L2** on the **Ylist**; highlight a **Mark**; **QUIT**.
- **GRAPH**.

Warm Ups

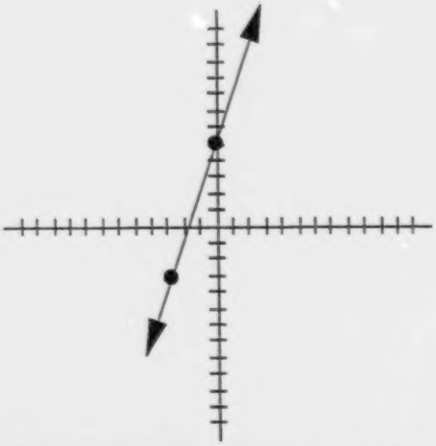
◆ Bill jogs at an average rate of seven miles per hour.

1. How many miles can he jog in two hours?
2. How many miles can he jog in three hours?
3. How many miles can he jog in x hours?
4. If Bill jogs 16 miles, how long has he been jogging?

◆ Find two values of x that make the equation $|x| = 7$ true. Find two values of x that make the equation $|x + 2| = 7$ a true statement.

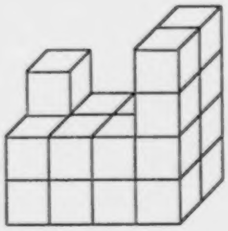
◆ What is the product of $2/8 \cdot 5/11 \cdot 8/14 \cdot \ldots \cdot 32/38$?

◆ Write an equation for the graph.



Challenge

This figure is built with cubes. The front, bottom, horizontal edge of the figure is six cm long. What is the volume of the figure? What is its surface area?



Week by Week Essentials

19

Writing Ideas

Explain how to solve **$2x + 7 = 23$**. Use your method to solve **$Ax + B = C$** for **x**.

Connections to the World

(Art) Ask each student to design a figure on the coordinate plane. Practice rotations, reflections, and translations around an axis. Identify, by coordinates, specific points on the original design and identify the corresponding points after each transformation. Ask students to find pictures in publications that have been transformed.

The WORDS are ...

monomial

binomial

trinomial

polynomial

Teacher to Teacher

Notes and Notebooks: There are many ways teachers can help students organize notebooks. Here are some suggestions.

◆ At the beginning of class, insist that notebooks be opened on desk.

◆ Show students samples of daily notes taken by students in previous classes.

◆ Have students write the daily objective at the top of their notes.

◆ Monitor the note taking process during class, with comments such as, "you can write this in your notes" and by walking around the class assessing student work.

◆ At the end of class, have students write a summary of the lesson in their notes.

◆ After a new lesson is introduced, students write responses to questions like the following. What were the goals for today's class? Why did I learn this? What strategies can I use to accomplish today's goals? What did I like best about today's class? What was frustrating about today's class? As part of their homework students are responsible for completing the questions and keeping the answers in a notebook.

◆ On a regular basis students will take during class a notebook quiz. Sometimes students are allowed to take these quizes with a partner. First they work on the quiz alone, a few minutes to talk together, and then put down their best responses on one piece of paper to be turned in from the pair.

Warm Ups

- Combine like terms.

a. (3 hamburgers, 4 fries, 5 drinks) + (2 hamburgers, 5 fries, 2 drinks)
b. $(9x + 3y + 7) + (2x - 8y + 9)$
c. $2\sqrt{3} + 7\sqrt{2} - 9\sqrt{3} + 8\sqrt{2}$
d. Do all the terms follow the same general rule? How would you explain the rule?

- What is the largest amount of money you can have in coins and still not have change for a dollar?

- Daniel buys pencils four for 25¢ and sells them three for 50¢. How many pencils must he sell to make $10?

- Represent the number 43 using seven 7s and any operations.

- Write each number 1 - 9 using each of the nine numbers 1 - 9 only once.

For example: $1 = (9 - 8)(7 - 6)(5 - 4)(3 - 2) \cdot 1$

Challenge

The sum of all of the integers between 50 and 350 which end in 1 is ... ?

Week by Week Essentials

20

Writing Ideas

Explain why a point on the x-axis has a y-coordinate of 0.

Connections to the World

(Transportation) Using a city map, ask the students to determine all of the ways to get from one location to another. Call local cab companies for rate information and find the cost of each of the trips to determine the cheapest trip.

The WORDS are ...

identify

classify

Teacher to Teacher

The Careless Checklist: In order to help students become more aware of careless mistakes and enable them to eliminate such mistakes, provide a checklist of the most commonly made ones. The list could include: computed wrong, used wrong operation, dropped a negative sign, did not distribute negative one, copied incorrectly, mistake in using a formula, did not use requested method, cancelled wrong, factored wrong, skipped the problem. The list would include mistakes made from inattention to details. The checklist would allow the student to record results from each assessment as it is returned and bring attention to any patterns that might appear. Credit could be given to the student for maintaining the checklist and reducing the incidence of careless mistakes.

Calculator Tips: Generating Random Numbers on the TI-81/82

- The **TI-81/82** will generate random numbers between 0 and 1.
- Go to the **HOME SCREEN (QUIT)**; key **MATH**; go to **PRB** menu; **1:rand**; **ENTER**. Continue to **ENTER** to generate a list of random numbers.
- When using the **RND** function for the first time with several calculators, you need to insure that each calculator generates a different sequence of random numbers. Follow these steps.
- Assign each calculator a different number; **QUIT**; key the number on the **HOME SCREEN**; **STO>**; **MATH**; go to **PRB** menu; **1:rand**; **ENTER**. Then generate random numbers using the previously mentioned procedure.

Warm Ups

- **Complete the table of values for $y = x^2 + 3$.**

x	y
-3	
-1	
0	
1	
2	

Graph the set of ordered pairs. Is the graph linear?

- **The numerator of a certain fraction is six less than the denominator. If the numerator is tripled and the denominator is increased by twelve, the value of the resulting fraction is 3/4. Find the original fraction.**

- **Find a number ending in eight that is the product of two consecutive whole numbers.**

Challenge

The bill for a lunch of three hamburgers and two drinks is $9.67. The bill for a lunch of four hamburgers and three drinks is $13.21. What is the total cost of one hamburger and one drink?

Writing Ideas

Describe any discoveries you have made during this week (chapter, unit, etc.) about mathematics or yourself doing mathematics.

Week by Week Essentials

21

Connections to the World

(Market Research) When big companies develop a new product, they spend much of their development funds on research. A variety of questions must first be answered before a product goes on the market. Consumers are often given surveys to complete that determine if a product is needed. Conduct a discussion on what questions you would ask and what data you would gather before marketing a product.

The WORDS are ...

coefficients

similar terms

Teacher to Teacher

Group Grading: Often teachers ask students to work together on projects or problems and give them a group grade. Some students resent this because they do not feel it is a fair practice. Here are some tips to help eliminate the fear of failure and unfair grading practices. Ask the students if they would prefer a group grade or individual grades. Be sure the criteria for grading is firmly established before the assignment begins for both group and individual grading. Get input from the students on what they believe is important in the grading of a problem or project. Make sure your groups are chosen with thoughtfulness and reflection.

Calculator Tips: **Tables for Your Function on the TI-82**

Using the **TABLE** function on the **TI-82**, one can view a function evaluated for integral values of **x**.

- **Y=**; key in the expression to be evaluated in **Y1**; **2nd WINDOW (TblSet)**; key in the values for **TblMin** and **ΔTbl** (increment); **2nd GRAPH (TABLE).**
- Scroll in the table using the up and down arrows. Notice integral values for **x** and the corresponding values of **Y1** are listed. The table can be expanded to include expressions for **Y2**, **Y3**, ...

Warm Ups

◆ Write each y-intercept in the corresponding square. The horizontal and vertical sums are the same.

a. $y = x - 2$
b. $y = x - 5$
c. $y = x - 8$
d. $y = 5x + 3$
e. $y = x - (1/4)x$
f. $y = 0.5x$
g. $y = 3x - 1$
h. $y = (1/2)x + 1/2$
i. $y = -0.5x + 0.5$

		a		
		b		
c	d	e	f	g
		h		
		i		

◆ Write the number 55 using five 4s.

◆ The speed limit along a particular highway increases from 55 mph to 65 mph, how much time will be saved on a 100 mile trip?

Challenge

Write 1/4 as the sum of two unit fractions. Write 1/5 as the sum of two unit fractions. Is there a pattern? Generalize.

Writing Ideas

Explain why an absolute value equation can have as many as two solutions.

Week by Week Essentials

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Connections to the World

(US Postal Service) The 1996 mailing rates for first class letters (up to 11 ounces) are 32¢ for the first ounce, then 23¢ for each additional ounce. However, any fraction of an ounce is rounded up to the next ounce. For instance, the rate for a letter weighing 3.4 ounces would be $.32 + 3 • .23 = $1.01. Material over 11 oz. changes to priority mailing rates. Have students draw graphs representing different postal rates, up to and beyond 11 ounces. Are the graphs functions? Check with USPS for current rates.

The WORDS are ...

interpret

analyze

Teacher to Teacher

Scoring Open-ended Problems: When scoring open-ended problems, activities, or projects use a rubric to evaluate the performance. Acquaint the students with it when an assignment is made. Consider using a variation of the following general rubric.

0 Does not address task, unresponsive, unrelated or inappropriate. Nothing correct.

1 Addresses item but only partially correct; something correct related to the question.

2 Answer deals correctly with more than one aspect of the question, but a significant portion is incorrect, missing, or unclear. May deal with all aspects but have major error(s).

3 Answer deals correctly with most aspects of the question, but something is missing. May deal with all aspects but have minor errors.

4 All parts of the question are answered accurately and completely. All directions are followed.

Warm Ups

◆ **Which is more favorable to the consumer: a discount of 15% followed by an increase of 15% OR an increase of 15% followed by a discount of 15%?**

◆ **Simplify**

a. $x + 2 - 3x$	e. $3x^2 - x^2 + 5$
b. $x^2 + 3x + 2x^2$	f. $(1/2)x^2 - (1/4) + (1/2)x^2$
c. $x - 3x + 5x$	g. $2.9x^2 + 0.5x - 1.8x^2$
d. $x^2 + 2x - x + 1$	h. $x^2 + x + 3.2x + 1$

◆ **Take any three digit number and write it down twice to make a six digit number. Why is the new number divisible by 7 or 11 or 13?**

◆ **What is the result of $2^{10} - 2^9 + 2^8 - 2^7 + ... + 2^2 - 2^1$?**

◆ **A month with five Wednesdays could begin on what day(s)?**

Challenge

The sum of the first eighty positive odd integers subtracted from the sum of the first eighty positive even integers is ... ?

Writing Ideas

Describe the relationship of the graphs of linear equations if **b** is the same for all but **m** is different and **b** is different but **m** is the same for all.

Week by Week Essentials

23

Connections to the World

(Science) Do you know why you see the flash of lightening before hearing the roll of thunder? Light travels at $1.86 \cdot 10^5$ miles per second while sound travels through air (at 32°F) in about 742 miles per hour. Compare the two velocities. Investigate how the velocities of the two phenomena are affected by factors such as altitude, temperature, and media.

The WORDS are ...

simplify

expand

factor

Teacher to Teacher

Managing Homework: Assign a number to each student in the class. At the end of class, call out two numbers. These two students are to turn in their homework to you the next morning before school begins. During planning time, break or lunch, make a copy of both homework papers for each group in the class. At the beginning of class, groups check their homework using the two student copies. Some advantages are:

- Understanding homework becomes students' responsibility. Students begin answering own questions.
- Students get practice in reading mathematics.
- Students writing homework solutions learn to write solutions carefully, show pride in their work, and become good peer models.
- The copy of worked solutions can be kept for anyone absent.

Calculator Tips: **Setting Zoom Factors (TI-81/82)**

- For the **TI-81: ZOOM**; **4:Set Factors**; enter values for **XFact** and **YFact**; **QUIT.**
- For the **TI-82: ZOOM**; go to the **MEMORY** menu; **4:SetFactors**; enter values for **XFact** and **YFact**; **QUIT.**

Warm Ups

◆ **A recent General Motors ad in the newspaper reads in part, "Our total worldwide sales of more than \$125 billion, including more than \$90 billion in the United States, was the highest in the world. GM sold more than 7.9 million cars and trucks ..."**
a. Write the numbers from the ad in expanded notation.
b. Write the numbers in scientific notation.
c. Which would you prefer? Why?

◆ **Multiply**

a. $(x + 2)(x - 5)$
b. $(2x + 3)(x - 6)$
c. $(x - 2)(x + 7)$
d. $(x + 8)(x + 2)$
e. $(2x + 1)(3x + 7)$

◆ **Julie purchased her stereo for \$620.10, including 6% sales tax. If she had bought the stereo while visiting her grandmother in a neighboring state where the sales tax is only 4%, how much money could she have saved?**

◆ **Find the difference between the sum of all even numbers and the sum of all odd numbers from 0 to 500.**

Challenge

If x is the sum of y, 4, 5, and 6 and $y = (1/4)x$, then what is x?

Week by Week Essentials

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Writing Ideas

Write a short paragraph that tells all that you know about the equation **$y = -2x + 7$**.

Connections to the World

(Geography) Ask each student to select a city from around the world. Determine the latitude and a particular day's temperature for each city. Graph latitude verse temperature for each city. Are there any noticable trends? Express any trends algebraically also. Temperature information is available regularly in larger daily newspapers.

The WORDS are ...

perfect square

perfect cube

Teacher to Teacher

Creating "Real World" Problems: Show how to take an article from the newspaper and create a "real-world" algebra problem. Have your students use newspaper and magazine articles to create similar problems. Compile, edit, and arrange the problems in useful formats. Select and use their work, with modification, to create homework/ classwork assignments, starter/challenge problems, labs/extended activities, and quiz/ test/exam items. An example is on **B-16**.

Calculator Tips: The Best-Fit Line (TI-82)

- Enter data.
- **STAT**; go to **CALC** menu; **5:LinReg(ax + b)**; **ENTER**. Calculator with display values for **a**, **b**, and **r**.
- The calculator uses the general form of a linear equation, **$y = ax + b$**.
- The correlation coefficient **r** **($-1 \le r \le 1$)** provides information about the quality of the linear fit.
- **Y=**; **VARS**; **5:Statistics**; go to **EQ** menu; **7:RegEQ**; **QUIT**.
- **GRAPH.**
- When finished, go to **STAT PLOT** and set plots to **Off**. **QUIT**.

Warm Ups

- Ann saved \$60 by buying a sofa at 20% off the regular price. How much did she pay for the sofa?

- Perform the indicated operations.

a. $2x \cdot x$	f. $-3x + x$
b. $x + x$	g. $-4x - 6x$
c. $2x + x$	h. $2x - 7x$
d. $-2(3)$	i. $3x \cdot 4x$
e. $-2(-3)$	

- What is the greatest area of a rectangle having a perimeter of 38 cm and sides of integral measure?

- What is the remainder when 5^{193} is divided by 7?

- Continue the patterns.

a. 1, 3, 6, 10, ____, ____, ____.
b. 2, 9, 16, 23, ____, ____, ___.

Challenge

If the number 99 is reduced by 10%, the resulting number is reduced by 20%, and the previous result is reduced by 30%, what is the final number?

Writing Ideas

Explain the advantages of writing a linear equation in the slope-intercept form.

Week by Week Essentials

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Connections to the World

(Social Studies) Compare the population density of selected countries with their standard of living. If a scale can be determined for standard of living, is there a correlation with population density? An American version for the 50 states would compare population density and average annual income.

The WORDS are ...

investigate

hypothesize

prediction

Teacher to Teacher

Test Rewrites: One method for taking care of partial credit easily.

- Grade tests the first time with no partial credit given.
- Return tests and give students time in class to talk in groups about their mistakes.
- Allow three to four days for them to find their mistakes and discover how to correct them.
- During that time, they cut out or copy the original problem and work it. Under it they must explain, in writing, why they missed the problem and what they could have done to arrive at a correct answer. Students receive no credit for simply reworking the problem.
- Up to 50% of the original value of the problem can be added to their original test grade as a result of the rewrite. More can be given for simple mistakes such as sign errors.
- It is true that a teacher handles a test paper twice during this process. However, they actually spend less time than usual in grading the papers. The teacher is no longer taking the time to diagnose errors and decide on uniform partial credit. Students are encouraged to take advantage of the opportunity to earn back points through the test rewrite process, but it is optional. Therefore, some students will not resubmit problems. Those that do are accepting more responsibility for their own learning and also learn that it is easier to do their work well the first time!

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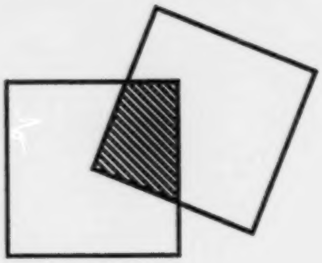
Warm Ups

◆ Simplify.

a. $10x \div 5x$	f. $44x^2 \div -4x^2$
b. $-24x \div 2x$	g. $3x^2 \div x$
c. $16x^2 \div 8x^2$	h. $-14x^2 \div 7$
d. $-10x^2 \div -x^2$	i. $32x^3 \div -8x$
e. $-4x^2 \div 2x^2$	

◆ How many whole numbers are between $\sqrt{10}$ and $\sqrt{95}$?

◆ One square rotates about a vertex fixed at the center of the other. What is the area of the shaded region?



◆ What is the hundreds digit for the sum 1! + 2! + 3! + ... + 10! ?

Challenge

Toni has nickels, dimes, and quarters. How many different ways can she make 50¢?

Week by Week Essentials

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Writing Ideas

Describe what slope means to you. Give some real world examples.

Connections to the World

(Physical Fitness) Ask the students to research records for high jump, long jump, mile run, and other activities. Graph the records over time and look for trends.

The WORDS are ...

restate

symbolic

represent

Teacher to Teacher

Resources for Data: Almanacs and the ***Statistical Abstract of the United States*** are excellent sources for numerical data covering a wide range of topics. The reference section of your library should have current editions of these publications. Newspapers generally are another good source for data and mathematics. The publishers of ***USA Today*** have an educational program called **Classline** which ties the newspaper with the academic subject areas. The mathematics program is called **How to Teach Math with USA Today** and the newspaper's educational development department has done its best to align the materials with the NCTM Curriculum Standards. For more information call 1-800-USA-0001. The **World Wide Web** can put you in touch with many sources for data as well as the most recent discoveries in science and mathematics. Once students are connected, the information is almost unlimited. The Department of Public Instruction's Web page is at **http://www.dpi.state.nc.us** .

Calculator Tips: Tracing on the Graph

Graph a curve on the **TI-81** (or **TI-82**). When you **TRACE**, the blinking cursor will be at the point of the curve with the x-coordinate in the center of the domain. In other words, the x-coordinate will be midway between **Xmin** and **Xmax**. Use the left and right arrow keys to move along the curve. If more than one curve is graphed, use the up and down arrow to change curves.

Warm Ups

◆ Classify the following numbers as rational or irrational.

a. -2.1	f. $-\sqrt{9}$
b. 9/5	g. -7
c. 2π	h. 12
d. 10 and 1/3	i. $3\sqrt{7}$
e. $\sqrt{5}$	j. -1/7

◆ The price of gold goes up 10% the first month, goes down 10 % the second month, and goes up 10% the third month. By what percent did the price of gold change from its initial price to the end of the third month?

◆ What are the sums of

$1 + 2 + 3 + \ldots + 99 + 100$?
$3 + 6 + 9 + \ldots + 297 + 300$?
$7 + 14 + 21 + \ldots + 693 + 700$?

Challenge

A coat is sale priced at $130 as the result of a 40% reduction in the original price. If the sale price is then reduced by 10%, the new price is what percent of the original price?

Week by Week Essentials

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Writing Ideas

Write a letter to an eighth grade (or pre-algebra) student telling them what they should know before tackling algebra.

Connections to the World

(Business) Discuss with your students the types of jobs that are paid on commission and how to find that commission. Interview people who hold those jobs and compare information.

The WORDS are ...

slope

vertical

horizontal

Teacher to Teacher

Tape It: If you are going to be out for a day or two for conferences or meetings, tape your lesson on video. Students love to see a tape of their teacher and you can be assured that the "right" things are being taught. If you are teaching a particularly difficult concept, tape it. Students can then review the tape at home or in the media center.

Calculator Tips: Graphing a Family of Equations on the TI-82

Graphing a family of equations, which differ in only one coefficient, can be accomplished in one line of the **Y=** menu by using { } around the changing coefficient. Graph $\mathbf{y = ax^2 + 4x + 2}$ where **a** takes on the integer values from -5 to 5. For **Y1** enter $\mathbf{\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}x^2 + 4x + 2}$, set your **RANGE**, and **GRAPH**.

Warm Ups

♦ In each equation x has the same value. Find the value of x.

a. $x(x - 5) = 0$
b. $x(y + 3) = 0$
c. $(x + 7)x = 0$
d. $3x + 5x = 0$

♦ Simplify.

a. $\sqrt{18}$
b. $\sqrt{12}$
c. $\sqrt{24}$
d. $\sqrt{72}$
e. $\sqrt{8}$
f. $\sqrt{20}$
g. $\sqrt{45}$
h. $\sqrt{81}$
i. $\sqrt{28}$

♦ The offical American flag has a length:width ratio of 1.9:1. A flag three meters long has stripes of what width?

♦ How many digits does $8^6 \cdot 5^{16}$ have when written in standard form?

Challenge

Find the area of the geometric shape formed from the intersection of $y \geq x - 2$, $y \geq -x - 2$, and $y \leq 1.3$. Explain how you solve the problem.

Writing Ideas

Explain how a pair of equations of lines can have no common point. Give an example.

Week by Week Essentials
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Connections to the World

(Radio and Television) Advertising supports radio and television broadcasting. Investigate how advertising rates are established, how they change over the years, and how they vary according to time-of-day. For example: In 1990 the cost of one commercial minute of prime time network television was $700,000; in 1991 the cost was $815,000; in 1992 $840,000; and in 1993 $850,000. Determine what the cost of a commercial minute would be in 1998. If half a minute of commercial time is 3/5 the cost of a whole minute, how many one and one-half minute commercials must be sold to gross $25,000,000?

The WORDS are ...

x-intercept

y-intercept

Teacher to Teacher

Bonus Points/Extra Credit: Many teachers award extra credit to students but are uncomfortable with traditional methods of working bonus points into students' grades. One method for doing this without over-inflating grades or awarding grades greater than 100 is illustrated in the following example. A teacher decides that bonus points accumulated while studying a unit are to be "worked into" the unit test grade. Before the unit test grade is determined, a student has correctly worked 83% of the test and has a total of 27 bonus points. The test grade is 100 • (83 + 27) ÷ (100 + 27) = 86. In general, **grade = 100 • (percentage + bonus points) ÷ (100 + bonus points)**. In this case it would take 14 bonus points to raise the 83 to 85 and 143 bonus points to raise 83 to 93. Using this method a student with 59% of the test correct would need 37 bonus points to get a grade of 70, 79 bonus points to get a grade of 77, 174 bonus points to get a grade of 85, and 486 bonus points to get a grade of 93. Obviously if many bonus points are awarded, any grade can rise substantially.

Warm Ups

♦ **Simplify, then complete the magic square with the results.**

a. $\sqrt{12}$
b. $\sqrt{75} - 2\sqrt{12}$
c. $4\sqrt{3} + \sqrt{12}$
d. $\sqrt{147}$
e. $\sqrt{27}$
f. $-2\sqrt{(3/4)}$
g. $\sqrt{121} - 11$
h. $\sqrt{363} - 3\sqrt{12}$
i. $(1/2)\sqrt{12} + \sqrt{27}$

a	b	c
d	e	f
g	h	i

♦ **In the first basketball game of the season, Becky made 7 field goals out of 20 attempts. In her next game Becky shot the ball 17 times. What is the fewest number of shots Becky could have made in the second game to raise her field goal percent above 50%?**

♦ **Simplify.** $(x - 2)^2 - (x + 2)^2$

Challenge

Write a system of inequalities whose solution is a diamond shape on the coordinate plane.

Week by Week Essentials

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Writing Ideas

Select one of the methods of solving a system of equations and write an argument for the method you select.

Connections to the World

(Sports) There are numerous statistics that describe how well a player or team is performing. In baseball, your batting average is determined by dividing the number of hits by the number of official times at bat. Choose teams or players with local interests in mind (better yet, let the students make the selections and do any research necessary) from which to create problems or projects. Example: If Chipper Jones has 30 hits and has batted 145 times, his average is 30/145 = .210. In the next 25 times at bat, what is the least number of hits he needs to raise his batting average to .250? .275? .290? .300?

The WORDS are ...

ratio

proportion

percent

Teacher to Teacher

Your Professional Organizations are Important: The North Carolina Council of Teachers of Mathematics (NCCTM) is a strong organization with membership of over 6,000 educators. Dues are minimal. As a member you receive the organization's magazine, The Centroid, and all of the programs for the fall and spring conferences. The conferences are excellent ways for continuing your education. While meeting and talking with colleagues you can glean activities and techniques to enhance your classroom experience. NCCTM, through its State Contest Committee, coordinates regional mathematics contests each spring. Students compete in four divisions: Algebra I, Geometry, Algebra II, and Comprehensive. State finals in Algebra I, Geometry, and Algebra II are held at three sites in each NCCTM region. The top 6% of the students from each Comprehensive Division site qualifies for the State Contest. Contest rules, eligibility requirements, and test sites are sent to middle and high school department chairs each fall. Further information can be obtained from the DPI mathematics staff or by writing to **NCCTM, P.O. Box 1783, Salisbury, NC 28145-1783**.

Warm Ups

◆ **Multiply and simplify.**

a. $(x + 5)(x - 3)$

b. $(\sqrt{x} + 2)(\sqrt{x} - 1)$

c. $(2\sqrt{x} + 5)(x - 6)$

d. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

e. $(2x - 1)(3x - 5)$

◆ **Find the sum.**

$1/2 + (1/2)^2 + (1/2)^3 + (1/2)^4 + \dots + (1/2)^{10}$

◆ **Two opposite sides of a square are increased in length by 20% while the other sides decrease by 35%. Find the percent change in the area of the figure.**

◆ **The sum of two numbers is 28. The product of the numbers is 7. Find the sum of the reciprocals of the numbers.**

Challenge

Jake won one million dollars on a television game show. He will be paid in the following manner. He will receive 1/3 of the prize money the first year, 1/6 the second year, 1/12 the third year, 1/24 the fourth year, and so on. How many years will it take to receive 50% of the prize money? 60%? 75%? 100%?

Week by Week Essentials

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Writing Ideas

List the major skills and concepts that were covered in the unit (chapter, unit, etc.). Include ideas that were new to you as well as any you already knew.

Connections to the World

(Motorsports, Automotive Technology) Collect and graph data about the winners of a particular automobile race (the Indianapolis 500 or NASCAR events at Charlotte and Rockingham are possibilities) and investigate trends. Research changes in technology and how that translates into performance on the raceway and on the street.

The WORDS are ...

linear

quadratic

exponential

Teacher to Teacher

Strategies for Opening Up Questions: Help you and your students better understand the mathematics being studied by opening up questions and asking for more than just the answer.

- When students have been using a particular operation, ask them to explain, in writing or with a diagram, what that operation means and how it works.
- With textbook problems add "Explain how you arrived at that answer".
- Instead of asking students to find THE answer, ask them to find out everything they can.
- Look for situations or rewrite existing problems so that they provide students opportunities to: explain their solutions, explain their reasoning, describe mathematical situations, write directions, create new problems, generalize a mathematical situation, and formulate hypotheses.
- Monitor the directions and responses given to students, so that whenever possible you can have them explaining to you rather than you explaining to them.

Warm Ups

- **Translate from words to symbols.**

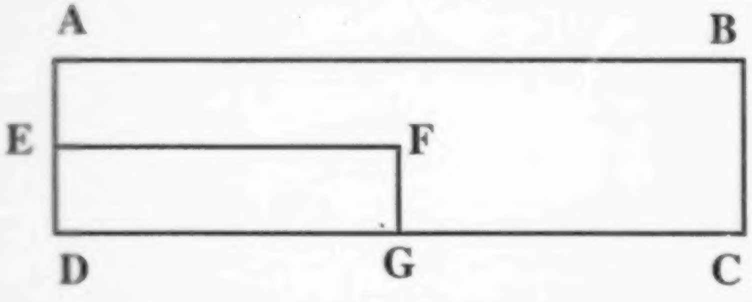
a. the sum of a number and seven
b. seven less than a number
c. the quotient of a number and three
d. ten more than a number
e. twice a number

- **If $x \copyright y = \sqrt{(xy)}$, find $(6 \copyright 54) \copyright 8$.**

- **Simplify $\sqrt{(2 \cdot 6 \cdot 15 \cdot 125)}$**

- **If the reciprocal of $(x + 1)$ is $(x - 1)$, then $x =$ ____ .**

- **If rectangle ABCD has an area of 90 yd^2 and E and G are the midpoints of sides AD and CD, then the area of DEFG is ...**



Challenge

Find the ordered pair of numbers (x,y) which satisfy $123x + 321y = 345$ and $321x + 123y = 543$.

Writing Ideas

If a new student started in this class today, what would you tell her about yesterday's class?

Week by Week Essentials

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Connections to the World

(Technology) Have students research major milestones in the development of calculating machines as well as the inventors of such machines.

The WORDS are ...

transformation

translation

reflection

rotation

symmetry

Teacher to Teacher

Bulletin Boards: If you are a bulletin board-challenged teacher, get students to assist in the creation of bulletin boards. Ask the students to bring in cartoons, poems, posters that are mathematics-related. Have students write down mathematics-related questions they have considered. Pin them to the board and have students write answers to the questions.

Calculator Tips: **The Best-Fit Exponential Equation (TI-81)**

- Enter data.
- **STAT**; **4:ExpReg**; **ENTER**. Calculator will display values for **a**, **b**, and **r**.
- The calculator uses the form of an exponential equation, $\mathbf{y = a \cdot b^x}$. The correlation coefficient **r** $(-1 \le r \le 1)$ provides information about the quality of the exponential fit.
- To graph the best-fit equation: **Y=**; **VARS**; go to the **LR** menu; **4:RegEQ**; **GRAPH**.
- To graph the best-fit equation and the scatter plot of the data: **STAT**; go to **DRAW** menu; **2:Scatter**; **ENTER**.

Warm Ups

- At $3 \cdot 10^5$ km/sec, how long does it take light to travel from the sun to Earth?

- With the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 listed in order from left to right, use the symbols (,), +, -, •, and ÷ to create an expression equal to 1000.

- Determine the minimum and maximum perimeter of a figure made of ten unit squares. When constructing the figures each square must share at least one side with at least one other square.

- If the ratio of a to b is 3 to 4 and the ratio of b to c is 2 to 1, then the ratio of a to c is what?

- A googol is 10^{100}. If one cubic centimeter of sand contains ten thousand grains of sand, how big is a pile containing googol grains of sand?

Challenge

What is the largest number of pieces into which you can cut a pizza which four straight cuts?

Writing Ideas

If extraterrestials landed today, how would you explain to them how to recognize irrational numbers?

Week by Week Essentials

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Connections to the World

(Driver's Education) Sometimes people estimate stopping distances for cars as a relationship of the number of car lengths per 10 mph. For example, you can estimate three car lengths for 30 mph and five car lengths for 50 mph. A more accurate approximation for stopping distances of cars is given by the equation $\mathbf{y = .071x^2}$. Create an equation to represent the estimations and investigate its relationship to the more accurate expression.

The WORDS are ...

standard forms

Teacher to Teacher

A Special Portfolio Review: Here are one North Carolina teacher's expectations for a student portfolio.

This semester each student will be required to keep a porfolio. A portfolio is a celebration of your best work. It will include a collection of your best homework, quizes, tests, and special projects as designated. Our portfolio review will be held on the evening of _________, from 4 to 8 PM. Your parents will be invited to review and grade your porfolios. Please start your collection today. Some of the items expected to be in the portfolio are listed below.

- your best homework/classwork assignments
- your best quiz papers, best test papers
- special projects: Career poster, Mathematician project
- Math Logos
- Pi Day
- Math Buttons (Display required on ______ in celebration of National Math Month.)
- All required items must be placed in your portfolio by April ____.
- Parents will review and evaluate your portfolio on April ____ between 4 and 8 PM.
- Parents will be asked to come at the time that is most convenient for them.

Please pay special attention to deadlines. Each project will be graded by the teacher and will count as he or she designates. The finished portfolio will count as a major test grade in the appropriate grading period. (See **B-17** for examples of a **Parent Letter** and an **Evaluation Sheet**)

Warm Ups

◆ Use only the numbers 2, 3, 6, and 9 to write four different proportions.

◆ Graph the exponents for the powers of 2 verses their corresponding unit digit. Describe the graph.

powers of 2	exponents	unit digit
2^0	0	1
2^1	1	2
2^2	2	4
2^3	3	8
2^4	4	6
.	.	.
.	.	.
.	.	.

◆ Find the perimeter of a square if the diagonal is 40 units in length.

◆ For what whole number N is the sum of N, N + 1, and N + 2 even?

◆ If the sum of two numbers is 16 and their product is 39, what is the sum of their squares?

Challenge

Simplify 1/7 • 4/10 • 7/13 • ... • 151/157 • 154/160.

Week by Week Essentials

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Writing Ideas

Explain the difference between x^2 and **2x**.

Connections to the World

(Transportation) Ask students to investigate how gears and their ratios affect the performance of bicycles and automobiles.

The WORDS are ...

formulate

derive

generalize

Teacher to Teacher

Homework: If students are turning in homework, have them write the amount of time they spent on the assignment and two to three sentences reflecting upon their progress and learning from the assignment. The reflections can increase communication with your students, emphasize process not just answers, and can help students self-evaluate their own learning.

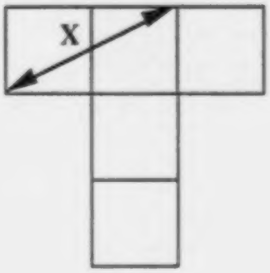
Rather than collecting all homework papers for a class, randomly check students work by row. Make a large die. Roll the die. If "one" is rolled, take up only homework for students in row one. Remind students to always do their homework, because they do not know when it will be taken up!

Calculator Tips: The Best-Fit Exponential Equation (TI-82)

- Enter data.
- **STAT**; go to **CALC** menu; highlight **A:ExpReg**; **ENTER** twice. Calculator will display values for **a**, **b**, and **r**.
- The calculator uses the general form of an exponential equation, $y = a \cdot b^x$.
- The correlation coefficient **r** $(-1 \le r \le 1)$ provides information about the quality of the exponential fit.
- **Y=**; **VARS**; **5:Statistics**; go to **EQ** menu; **7:RegEQ**; **QUIT**.
- **GRAPH;** when finished, go to **STAT PLOT** and set plots to **Off**; **QUIT**.

Warm Ups

◆ Five congruent squares are arranged in a T-shape as shown. If X = 15, then find the area of the T-shaped figure.



◆ Find the sum.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{98 \cdot 99} + \frac{1}{99 \cdot 100}$$

◆ If p and q are negative numbers, which of the following are always negative?

a. $p + q$ b. $p \cdot q$ c. $p - q$

Challenge

Justin enters a game with a .283 batting average. After going 4-for-5 in the game, he has a .305 batting average. How many times had he batted before the game began?

Week by Week Essentials

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Writing Ideas

Write a study guide for reviewing for an upcoming test.

Connections to the World

(Earth Science, Social Studies) Using the appropriate references, track the consumption of a particular natural resource, such as petroleum, over a period of years. How has the availability of that resource affected political and social decisions in the United States ?

The WORDS are ...

vertex

minimum

maximum

Teacher to Teacher

Review Cards with State Objectives: Select sample problems from the state objectives and other items that you know match the state objectives. Copy each problem to an index card. Use a different color of cards for each goal. If you want to review goal 5, pull out the "blue" cards. Write the solutions to the problems on the back of the card. Use the cards for warm ups, closure, reviews, or with a game.

One such game is football, played with four students, two on a team. Students draw a football field on a piece of paper and each team begins at opposite 30 yard lines. Paper clips can be used for markers. One pair draws a card, the students work the problem, and if they get the solution correct they gain 10 yards. If they do not answer the problem but the opponents do, they lose ten yards. If the opponents cannot answer the questions, they neither gain nor lose points. The opponents then draw a card, if they answer correct, they move their football 10 yards. A team receives 7 points for crossing the goal line.

Calculator Tips: The Best-Fit Quadratic Equation (TI-81)

The TI-81 does not have a quadratic best-fit function. On **B-90** there is a calculator program that students can enter in their calculator and use. Directions for running the program are included.

Warm Ups

- At the one-mile Phoenix International Raceway, the winning vehicle in Saturday's Supertruck Race had an average speed of 91 mph. On Sunday the stock cars raced and the winner averaged 102 mph. In seconds, how much longer does it take the winning truck to complete a mile compared to the winning stock car?

- Make 500 using only 8s.

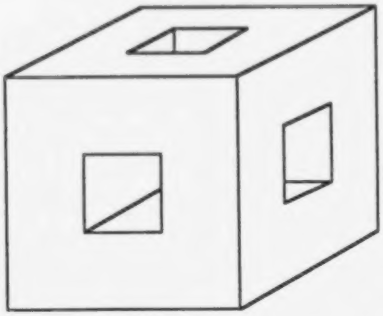
- Solve for x.

$$7^{(x^2 - 25)} = 1$$

- If (a + b) ÷ a = 2 and (c + b) ÷ c = 3, what is the value of a/c?

Challenge

The plastic cube shown originally had a volume of 1500 cm³. The front face is drawn to proportion. Square holes were cut through to the opposite face. What is the volume of the remaining plastic?



Week by Week Essentials

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Writing Ideas

Bill had to calculate the tip on the bill at Shoney's for his friends. The bill is $26.40. Explain a quick way Bill can determine a 15% tip without using paper and pencil.

Connections to the World

(Sports) Pick a team, baseball, basketball, football, etc. Pick a statistic and follow it over time. The Olympics provide a rich source of data for investigations into changes in individual athletic performance. Look for trends. Discuss reasons for changes. See **Olympic Swimming (B-66)** and **Scoring and Winning (A-7)** as examples.

The WORDS are ...

real

rational

irrational

Teacher to Teacher

History of Algebra: Algebra is a Latin variation of the Arabic word ***al-jebr***, from the title of the ninth century mathematics book ***Al-jebr w'almaqabala***, by the Arab mathematician Mohammed ibn-Musa al-Khowarizmi. The title literally means "restoration and reduction". Restoration was the transposition of negative terms to the opposite side of an equation and reduction was the simplification of similar terms.

There are a number of historical figures worthy of exploration by algebra students. Consider these (and of course others) as suitable for class discussions, papers, projects, extra credit, enrichment, and/or quiz and test items. See **B-18** for an outline illustrating how a mathematician project can be laid out for students.

al-Khowarizmi
Aryabhata
Girolamo Cardano
Diophantus
Carl Fredrich Gauss
Omar Khayyam
Pythagoras

Apollonius
Bhaska
Chhin Chiu-shao
Euclid
William George Horner
Mahavira
Niccolo Tartaglia

Archimedes
Brahmagupta
Rene Descartes
Fibonnacci
Abu Kamil
Blaise Pascal

Warm Ups

◆ In the year he was elected president, had Thomas Jefferson invested \$25 in a bank account that paid 6% interest compounded annually, how much money would be in the account by 2000?

◆ If the digits of a three digit number are added and cubed, the result is the original number. What is the number?

◆ At what temperature are the Fahrenheit and Celcius readings on a thermometer the same? Remember $F = (9/5)C + 32$.

◆ Linda was driving home from the football game. Because of the heavy traffic, she was only able to travel at a rate of 25 mph for the first three miles. For the remaining 14 miles to her house she was able to travel at 50 mph. For the entire trip, what was her average speed?

Challenge

The graphs of $2y + 3 + x = 0$ and $3y + ax + 2 = 0$ are lines. If the lines are perpendicular, find the value of a.

Writing Ideas

Write a letter describing difficulties you are having with a particular mathematical concept.

Week by Week Essentials

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Connections to the World

(Financial Planning) Investors often are told "you can double your money at the interest rate of 8% in only 9 years". Students can investigate the number of years it takes to double an investment, investigating patterns with $\mathbf{(1 + i/100)^n}$. Using a calculator, by trial and error, students can simplify expressions such as $(1.06)^{10}$ to see if the investment would double. Have students make a table with the interest rate and the years to double. Is there a pattern? In the financial market, the rule of 72 is used to describe this pattern. If you divide 72 by the interest rate, you can approximate the number of years it would take for the investment to double.

The WORDS are ...

compare

contrast

Teacher to Teacher

Curving Grades: At various times in a teacher's career it becomes necessary to curve grades. The statistician would use z-scores and adjust the set of scores to a normal curve. Fortunately there are less numerically intensive methods for doing the job. One method uses the formula, **adjusted score = 100 - F(100 - original score)**, where **F** is a scale factor. A scale factor of 0.8 converts 80 to 84 and 50 to 60. The smaller the scale factor the greater the scaling effect. Another quick method is to take the **square root** of the original score and multiply the result by **ten**.

Calculator Tips: The Best-Fit Quadratic Equation (TI-82)

- Enter data.
- **STAT**; go to **CALC** menu; **6:QuadReg**; **ENTER**. Calculator with display values for **a**, **b**, and **c**.
- The calculator uses the general form of a quadratic equation, $\mathbf{y = ax^2 + bx + c}$.
- **Y=**; **VARS**; **5:Statistics**; go to **EQ** menu; **7:RegEQ**; **QUIT**.
- **GRAPH**; when finished, go to **STAT PLOT** and set plots to **Off**; **QUIT**.

Warm Ups

◆ What is the value of c if the vertex of the parabola $y = x^2 - 8x + c$ is a point on the x-axis?

◆ Find the number that is less than 100, an odd number, a multiple of five, divisible by three, and digit sum is an odd number.

◆ For all integers x,

$\underline{X} = x$ if x is positive or zero
and
$\underline{X} = x + 1$ if x is negative.

Find $-\underline{10} + \underline{10}$.

◆ If $3! = 1 \cdot 2 \cdot 3$ and $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$, then what does $89!/90!$ equal?

◆ How many integers greater than 10 and less than 100 are increasd by 9 when their digits are reversed?

Challenge

A vacuum pump is designed so that on each stroke it removes 5% of the gas in the chamber. How many strokes are necessary to remove 99% of the gas that was present when the pump began operation?

Answers You Have Been Looking For

For many of the problems presented in the *Warm Ups* and *Challenges* there can be more than one correct answer. Always check or let the students check one another's answers. Encourage students to explain how they arrived at their result. Many times students will come up with results that are correct that you or the authors of this publication have not thought of. As best we can we will indicate problems where we expect other correct answers.

Week 1

Warm Ups:

- a. beginning with any vertex, 6-1-5-3-4-2.
  b. beginning with any vertex, 1-5-2-4-3-6.
  c. beginning with any vertex, 5-1-4-2-6-3.
  d.
  e. beginning with any vertex, 1-2-3-6-5-4.
- one arrangement that works is 127 + 368 = 495
- beginning with any vertex,
  9 - 3 - 4 - 7 - 6 - 2 - 8 - 1 - 5.

Challenge:

- 1/2

Week 2

Warm Ups:

- -6, 8, -12, -13, 7.8
- 31
- 1. 2
  2. -11
  3. 2.5
  4. 8
  5. -4
- a. 0
  b. -1
  c. 4
  d. 5
  e. 1
  f. -3
  g. -2
  h. 3
  i. 2

Challenges:

- 6
- 143

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Week 3

Warm Ups:
- 12, 13, 14, 15, 16, 17
- 45, 46, 47
- 7
- 24
- 450

Challenges:
- $244 + 244 + 274 = 752$
- $21978 \cdot 4 = 87912$

Week 4

Warm Ups:
- 2, 3, -1, -1, 1, 2, -2, 1
- (Quarters, Nickels) (14, 1), (13, 6), (12, 11), or (11, 16)
- 3^{40}
- 2^{77}
- $(3967 \cdot 2) \cdot 50 =$ $= 3967 \cdot 100 = 396700$

Challenges:
- 2454219
- 40

Week 5

Warm Ups:

- 15 cm^2, 396 cm^3, 15 cm
- 1. -8 3. 5
  2. 4.41 4. -4.3
- 378 cm^3
- 20
- 3^{75}; $(2^4)^{25} < (3^3)^{25}$

Challenge:
$21 + 6 = 27$

Week 6

Warm Ups:

- 1. T 4. F
  2. T 5. T
  3. F 6. F
- 1. -2 5. 12
  2. -21 6. 70
  3. 5 7. -12
  4. 21 8. 20
- 336.96 cm^2

Challenge:
$(3/4)^4 = 81/256$

Week 7

Warm Ups:
- 1. no 4. yes
  2. yes 5. yes
  3. no 6. yes
- 63, 27, 31, second, order of operations
- 172
- 16/45

Challenge:
some possibilities are:

$0 = (1 - 9/9) \cdot 7$
$1 = 1 + (9 - 9) \cdot 7$
$2 = 1^7 + 9/9$
$3 = (9 + 9) \div (7 - 1)$
$4 = \sqrt{(9 + 7)} \cdot 1^9$
$5 = \sqrt{(9 + 7)} + 1^9$
$6 = 7 + 9 - 9 - 1$
$7 = 7 \cdot 1 + (9 - 9)$
$8 = 7 + 1 + 9 - 9$
$9 = 9 \cdot 1^{(9 + 7)}$
$10 = 9 + 1^{(9 + 7)}$
$11 = 7 + \sqrt{9} + 1^9$
$12 = 9 + 7 - \sqrt{9} - 1$
$13 = 9 \cdot 1 + 7 - \sqrt{9}$
$14 = 1 + 9 + 7 - \sqrt{9}$
$15 = 9 + 7 - 1^9$
$16 = 9 + 7 \cdot 1^9$
$17 = 9 + 9 - 1^7$
$18 = (9 + 9) \cdot 1^7$
$19 = 9 + 9 + 1^7$
$20 = 7 + 9 + 1 + \sqrt{9}$
$21 = 7 \cdot \sqrt{9} \cdot 1^9$
$22 = 7 \cdot \sqrt{9} + 1^9$
$23 = 7 \cdot \sqrt{9} + \sqrt{9} - 1$
$24 = 9 + 9 + 7 - 1$
$25 = 1 \cdot 9 + 9 + 7$

Week 8

Warm Ups:

- follow the algebra
  x -- $4x$ -- $4x + 4$ --
  $x^2 + 4x + 4$ -- $x + 2$ -- 2
- $96
- 49, 50
- at least 11

Challenge:
there are ten: 888, 789, 798, 879, 897, 978, 987, 699, 969, 996

Week 9

Warm Ups:

- area, base, height
  $h = A/b$, 12 feet
- 1. 3 3. -5
  2. 4 4. -5
- 11
- 272 cm

Challenge:
one possibility is
$7 \cdot (7 - 7/7) + (7/7)^7$

Week 10

Warm Ups:

- 1. $x = 16$
  2. $x > 16$
  3. $x < -5$
  4. $x < 16$
  5. $x < 5$
  6. $x > 5$
- $C = 2.20\,N$
- 1024
- 18

Challenge:
6, 30, 54, 60, 84, 96, 120, 150

(9/3/96) Computer scientists at Silicon Graphic's Cray Research Unit have discovered a large prime number while conducting tests on a CRAY T90 supercomputer. The number is $\mathbf{2^{1257787} - 1}$, 378,632 digits long and would occupy 12 newspaper pages. See this web site for more information: http://reality.sgi.com/csp/ioccc/noll/prime/prime_press.html

Week 11

Warm Ups:

- 318, 319
- 7.8
- 6, 91
- 1. 9/20 3. 9/11
  2. 11/20 4. 20/11

Challenges:

- 7
- 4

Week 12

Warm Ups:

- 5
- possibilities are
  (1, 3), (-2, 2), (-3, 1)
  above the line
- 3
- $4^{60}, 3^{72}, 5^{48}$
- 91

Challenge:
answers will vary
$(9/9)^9, (9/9)^{9+9}, (9/9)^{9+9+9}$
use the same arrangements regardless of the number

Week 13

Warm Ups:

- $3x + 5x = 120$
  $5A + 30 = 3$
  $45y + 10 = 12$
- -7/8
- $n = 5/3$
- 2.5, 2.8
- after traveling 100 miles in 2 hr, the traveler stops for 2 hr, and then travels 50 more miles in 1 hr.

Challenge:
10 coins; 3 quarters, 2 dimes, 1 nickel, 4 pennies

Week 14

Warm Ups:

- 1. 7
  2. 9
  3. -4
  4. -17
  5. -4
  6. 4
- no; 80%, no; 87%, no; 93%, better
- pentagon
- 1. T, commutative
  2. T, associative
  3. T, associative
  4. T, commutative
  5. F
  6. F
  7. T, distributive

Challenge:
39

Week 15

Warm Ups:

- 5, 10, 15, ..., 35
- $625 = 25^2$
- 1, 3, or 5
- a. 2
  b. -1
  c. 14
  d. 17
  e. 5
  f. -7
  g. -4
  h. 11
  i. 8

Challenge:
350

Week 16

Warm Ups:

- 1. add -7
  2. add 9
  3. add -y
  4. add 2y
  5. add -3y, multiply/divide by -1
  6. multiply by -1/2, add -y
- 4
- 1. $2(x + 2) = 12$
  2. $2(x + x - 2) = 20$
  3. $2(2x + x) = 18$
  4. $2(x + 4 + x) = 88$
- 0
- 1996^{1997}

Challenge:
$41^2 + 43^2 + 45^2 = 5555$

Week 17

Warm Ups:

- $1 = 1$
  $2 = 1 + 1$
  $3 = 1 + 1 + 1$
  $4 = 4$
  $5 = 4 + 1$
  $6 = 4 + 1 + 1$
  $7 = 4 + 1 + 1 + 1$
  $8 = 4 + 4$
  $9 = 9$
  $10 = 9 + 1$
  $11 = 9 + 1 + 1$
  $12 = 9 + 1 + 1 + 1$
  $13 = 9 + 4$
  $14 = 9 + 4 + 1$
  $15 = 9 + 4 + 1 + 1$
  $16 = 16$
  $17 = 16 + 1$
  $18 = 16 + 1 + 1$
  $19 = 16 + 1 + 1 + 1$
  $20 = 16 + 4$
  $21 = 16 + 4 + 1$
  $22 = 16 + 4 + 1 + 1$
  $23 = 16 + 4 + 1 + 1 + 1$
  $24 = 16 + 4 + 4$
  $25 = 25$

- $1 = 2^0$
  $2 = 2^1$
  $3 = 2^1 + 2^0$
  $4 = 2^2$
  $5 = 2^2 + 2^0$
  $6 = 2^2 + 2^1$
  $7 = 2^2 + 2^1 + 2^0$
  $8 = 2^3$
  $9 = 2^3 + 2^0$
  $10 = 2^3 + 2^1$
  $11 = 2^3 + 2^1 + 2^0$
  $12 = 2^3 + 2^2$
  $13 = 2^3 + 2^2 + 2^0$
  $14 = 2^3 + 2^2 + 2^1$
  $15 = 2^3 + 2^2 + 2^1 + 2^0$
  $16 = 2^4$
  $17 = 2^4 + 2^0$
  $18 = 2^4 + 2^1$
  $19 = 2^4 + 2^1 + 2^0$
  $20 = 2^4 + 2^2$
  $21 = 2^4 + 2^2 + 2^0$
  $22 = 2^4 + 2^2 + 2^1$
  $23 = 2^4 + 2^2 + 2^1 + 2^0$
  $24 = 2^4 + 2^3$
  $25 = 2^4 + 2^3 + 2^0$

(see next page)

Week 17 (cont.)

- 1. $3x - x < -12$ 3. $3x + 1 < 2x$
  2. $2x + 3 > 43$ 4. $5x - 1 > 15$
- \$22.20, distributive property

Challenge:

$13^2 - 12^2 = 5^2$

Week 18

Warm Ups:

- 1. 14 2. 21 3. $7x$ 4. 2.3 hours
- -7, 7; -9, 5
- 1/133
- $y = (8/3)x + 5$

Challenge:

70.875 cm^3, 126 cm^2

Week 19

Warm Ups:

- a. 5 hamburgers, 9 fries, 7 drinks
  b. $11x - 5y + 16$
  c. $15\sqrt{2} - 7\sqrt{3}$
  d. combine like terms
- \$1.19; 3 quarters, 4 dimes, 4 pennies
- 96
- one arrangement $7 \cdot (7 - 7 \div 7) + (7 \div 7)^7$
- some possibilities are
  $1 = (9 - 8)(7 - 6)(5 - 4)(3 - 2) \cdot 1$
  $2 = (9 - 8)(7 - 6)(5 - 4)(3 - 2) + 1$
  $3 = (9 - 8)(7 - 6)(5 - 4)(2 - 1) \cdot 3$
  $4 = (9 - 8)(7 - 6) \cdot 4 \cdot 1 + (5 - 3 - 2)$
  $5 = (9 - 8)(7 - 6)(4 - 3)(2 - 1) \cdot 5$
  $6 = (9 - 8) \cdot 6 \cdot (4 - 3) + (7 - 5 - 2) \cdot 1$
  $7 = (9 - 8)(6 - 5)(4 - 3)(2 - 1) \cdot 7$
  $8 = 8 \cdot (7 - 6)(3 - 2) \cdot 1 + (9 - 5 - 4)$
  $9 = 9 \cdot (8 - 7)(6 - 5)(4 - 3)(2 - 1)$

Challenge:

5880

Week 20

Warm Ups:

- 12, 4, 3, 4, 7; the graph is not linear
- 6/12
- none

Challenge:

\$3.54;
$$\begin{array}{r} 4h + 3d = 13.21 \\ -\ \underline{(3h + 2d) = \ 9.67} \\ 1h + 1d = \ 3.54 \end{array}$$

Week 21

Warm Ups:

- a. -2
  b. -5
  c. -8
  d. 3
  e. 0
  f. 0
  g. -1
  h. 1/2
  i. 0.5
- one possibility is $44 + 44/4$
- about 17 minutes

Challenge:

$1/4 = 1/5 + 1/20$
$1/5 = 1/6 + 1/30$
$1/n = 1/(n+1) + 1/[n(n+1)]$

Week 22

Warm Ups:

- the same
- a. $-2x + 2$
  b. $3x^2 + 3x$
  c. $3x$
  d. $x^2 + x + 1$
  e. $2x^2 + 5$
  f. $x^2 - (1/4)$
  g. $1.1x^2 + 0.5x$
  h. $x^2 + 4.2x + 1$
- 1001, which is 7 • 11 • 13, is a factor of the new number
- 682
- Monday, Tuesday, Wednesday

Challenge:
80

Week 23

Warm Ups:

- a. 125,000,000,000
  90,000,000,000
  7,900,000
  b. $1.25 \cdot 10^{11}$, $9 \cdot 10^{10}$, $7.9 \cdot 10^{6}$
  c. answers will vary
- a. $x^2 - 3x - 10$
  b. $2x^2 - 9x - 18$
  c. $x^2 + 5x - 14$
  d. $x^2 + 10x + 16$
  e. $6x^2 + 17x + 7$
- \$11.70
- 250

Challenge:
20

Week 24

Warm Ups:

- \$240
- a. $2x^2$
  b. $2x$
  c. $3x$
  d. -6
  e. 6
  f. $-2x$
  g. $-10x$
  h. $-5x$
  i. $12x^2$
- 90 cm^2
- 5
- 15, 21, 28
  30, 37, 44

Challenge:
49.896

Week 25

Warm Ups:

- a. 2
  b. -12
  c. 2
  d. 10
  e. -2
  f. -11
  g. $3x$
  h. $-2x^2$
  i. $-4x^2$
- 6
- 1/4 of the square
- 9; 4,037,913

Challenge:
10

Week 26

Warm Ups:

- a. rational
  b. rational
  c. irrational
  d. rational
  e. irrational
  f. rational
  g. rational
  h. rational
  i. irrational
  j. rational
- 8.9%
- 5050
  15150
  35350

Challenge:
54%

Week 27

Warm Ups:

- 0
- a. $3\sqrt{2}$
  b. $2\sqrt{3}$
  c. $2\sqrt{6}$
  d. $6\sqrt{2}$
  e. $2\sqrt{2}$
  f. $2\sqrt{5}$
  g. $3\sqrt{5}$
  h. 9
  i. $2\sqrt{7}$
- 0.12 m
- 17

Challenge:
10.89 units^2

Week 28

Warm Ups:

- a. $2\sqrt{3}$
  b. $\sqrt{3}$
  c. $6\sqrt{3}$
  d. $7\sqrt{3}$
  e. $3\sqrt{3}$
  f. $-\sqrt{3}$
  g. 0
  h. $5\sqrt{3}$
  i. $4\sqrt{3}$
- 12
- $-8x$

Challenge:
many possibilities

Week 29

Warm Ups:

- a. $x^2 + 2x - 15$
  b. $x + \sqrt{x} - 2$
  c. $5x - 12\sqrt{x} + 2x\sqrt{x} - 30$
  d. $a - b$
  e. $6x^2 - 13x + 5$
- 1023/1024
- 22% decrease in area
- 4

Challenge:
50% in two years; 60% in fours years; the winner will never receive more than 2/3 of the prize money

Week 30

Warm Ups:

- a. $x + 7$
  b. $x - 7$
  c. $x \div 3$
  d. $10 + x$
  e. $2x$
- 12
- 150
- $\pm\sqrt{2}$
- 22.5 yd^2

Challenge:
(1.5, 0.5)

Week 31

Warm Ups:

- 8 minutes
- one possibility is
  $(1 + 2) \bullet 3 + (4 \bullet 5)(6 \bullet 7 + 8) - 9$
- minimum = 14
  maximum = 22
- 3:2
- 10^{81} km^3

Challenge:
11

Week 32

Warm Ups:

- $2/6 = 3/9$; $2/3 = 6/9$;
  $3/2 = 9/6$; $6/2 = 9/3$
- periodic, repeating pattern
- $8\sqrt{5} \approx 17.9$
- when N is odd
- 178

Challenge:
$4/(157 \bullet 160) = 1/6280$

Week 33

Warm Ups:
- 225
- 99/100
- p + q

Challenge:
113

Week 34

Warm Ups:
- 4.3 seconds
- one possibility is $8 \cdot 8 \cdot 8 - 8 - (8 + 8 + 8 + 8) \div 8$
- ±5
- 2

Challenge:
1111 and 1/9 cm^3

Week 35

Warm Ups:
- $2,878,147.60
- 512
- -40°
- 42.5 mph

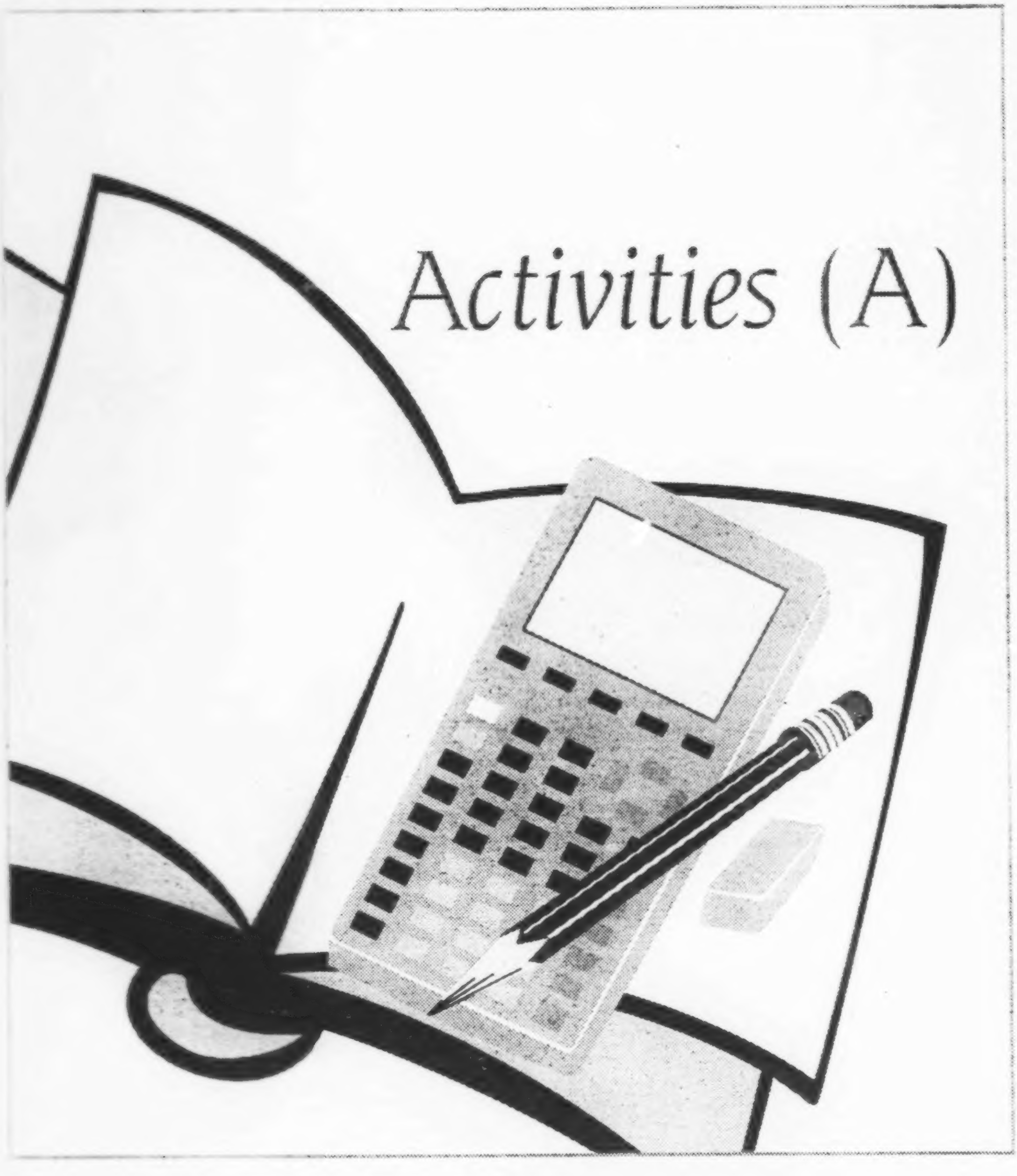
Challenge:
-6

Week 36

Warm Ups:
- 16
- 45
- 1
- 1/90
- 8

Challenge:
90

Activities (A)



Lining Up Dominoes

Concepts and Competencies

Focus and Review: Simplify real number expressions, apply the distributive property.

The puzzle format used in this activity can also be used to address: evaluating algebraic expressions, using algebraic properties, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, operating with polynomials, and solving quadratic equations.

Goal

Simplify real number expressions and apply the distributive property in an interesting format.

Materials

One set of dominoes (**B-97, 98**) per pair (or team) of students, blank domino sheets (**B-99**).

Description

Students will make a train of dominoes by successfully simplifying an expression or applying the distributive property. Blank domino sheets (**B-99**) can be made available so that students can create versions of the game that practice various algebraic skills throughout the year. **Well-constructed and edited student versions of dominoes will provide the teacher a pool of materials to use thereafter.**

Procedure

One person shuffles the dominoes and deals an equal number to each player.

Students must start with the playing piece that says "Start" or "Begin".

Students try to connect dominoes together by simplifying the expression on one domino to get an answer on another domino.

Play continues until all pieces are connected. (Prizes, extra credit, etc. can be awarded for teams or pairs that finish first.)

Copy the blank sheet (**B-99**) for students to create other versions of dominoes.

I Have ... Who Has ...

Concepts and Competencies

Focus:	Perform operations with integers. Perform operations with polynomials.
Review:	Use the associative, commutative, and distributive properties.

Goal

Using paper and pencil students will rapidly perform operations with polynomials or integers. Students will use the format to create their own versions of the activity.

Materials

Two copies of the blackline masters (**B-100, 101** or **B-102, 103**), one separated.

Description

Students will listen, perform operations, and respond when appropriate in a round-robin format. Students will need to be able to complete operations with polynomials using paper and pencil.

Procedure

Each student will be given a piece of paper with a response (**I Have ...**) and directions (**Who Has ...**) for the next student. Pass out all the slips. Students may have more than one.

I Have ...	Who Has ...
$x + 4$	**my expression multiplied by 4**

Choose any student to start. Have that student stand and read distinctly their slip.

All students should mentally or on paper complete the indicated operations.

The student with the correct response (**I Have ...**) should stand and read their slip.

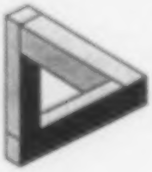
This same sequence continues until the correct response returns to the first slip.

The teacher or student facilitating the activity should follow the progress of responses on the unseparated master sheet.

Using the blank page (**B-104**), students can create their own versions of the activity as a homework assignment or for extra credit. **Well-constructed and edited student versions of the activity will provide the teacher a pool of materials to use thereafter**.

With several versions of ***I Have ... Who Has ...*** a teacher is able to separate the class into smaller groups to do the activity.

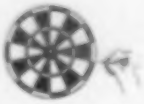
Scientifico



Concepts and Competencies

Focus: Scientific notation

Review: Place value, decimals, exponents



Goal

Students will be able to translate scientific notation numbers into standard form.

Materials

Gameboard (**B-105**), 6 markers of one color for each student

Description

Students practice translating scientific notation numbers into standard notation using the game **Scientifico**. Students take turns rolling three dice and constructing a number in scientific notation. Ex: 3, 6, 4 can be written 3.6×10^4. After recording this number on the recording chart, the student places a marker in the proper place on the game board. The student who can make three numbers in a row, column, or diagonal is the winner.

Procedure

The game can be played in pairs. Decide whether you wish students to play using positive or negative exponents.

Give each pair of students a gameboard (**B-122**), three dice, and 6 colored markers for each student. (Or the teacher can lead the game by rolling the dice for the class)

A student rolls three dice and uses the outcomes to make a number in scientific notation.

The student then writes the number in both scientific form and standard form on a recording chart.

The student places a chip in the proper place on the game board.

Ex: 2, 3, 5

3.5×10^2

Number in Scientific Notation	Standard Form
3.5×10^2	350
6.3×10^6	6,300,000

The second student rolls the dice, writes a number in scientific notation, and records both forms.

The student also places a marker on the board. If a student is unable to make a number on the board, his or her turn passes.

The game continues until a student has three markers in a row, column, or diagonal.

Scoring and Winning

Concepts and Competencies

Focus:	Graph a relation on the coordinate plane.
Review:	Find and use linear equations.

Goal

Among the statistics that are collected in team sports, scoring is the one that most often correlates with winning. **(cause and effect?)** In team sports there are two categories of scoring, offense (points scored) and defense (points allowed). Which category correlates best with winning?

Materials

Calculators, NFL scoring statistics (**B-106**).

Description

Students will use the data from the NFL (or similar data from the NBA, NHL, MLB, or local minor leagues) to create scatter plots and find lines of best fit. Students will discuss the characteristics of those lines and make predictions.

Procedure

Divide the class into two groups. One group will be responsible for offense and the other defense.

Have each group enter the set of ordered pairs (scoring, wins or losses) into the calculator and graph. Describe the scatter plot. Determine the correlation coefficient (a number between -1 and 1) for each set. The correlation coefficient is one indicator of how well the data resembles a linear relationship.

Discuss the results and write a brief description.

Discuss independent and dependent quantities. Do wins depend upon scoring or does scoring depend upon winning?

Calculate the average scoring margin for each team (offense minus defense). **What does a negative scoring margin mean?**

Have the class enter the set of ordered pairs (scoring margin, wins) into the calculator and graph. Determine the correlation coefficient for that set of data. Discuss the results.

Have students graph by hand their data (offense, defense, or scoring margin). Then using a straight edge fit a line in the set of ordered pairs. Have several students use a transparency of the coordinate grid to plot their data. Then at the overhead place or draw a line "fitting" the ordered pairs.

If a football team wins 14 games, how many points should it average on offense? defense?

By how many points should the team win?

Which of these is the most accurate prediction? Why?

What about the team that goes undefeated (16 wins)? No wins?

When the students are ready, use the graphing calculator to its fullest.

Use the calculator to determine the line of best fit for each set of data and graph with the ordered pairs. Discuss the graphs.

Discuss the slope, and y-intercept of each linear equation within the context of the data.

Gulliver's Clothes

Concepts and Competencies

Focus: Graph and compare relations in a real-world setting.

Review: Graph ordered pairs in the coordinate plane.

Goal

By comparing a scatterplot designed by using data collected by the students to the linear function **y = 2x**, students will be able to make predictions and justify their predictions.

Materials

Measuring tapes, data recording sheets (**B-107**), graphing calculators.

Description

This activity will have students working in small groups collecting and recording the circumference of each group member's thumb, wrist, and neck. The data will be plotted on a scatterplot and compared to a linear function. Students will use the scatterplot and linear function to make predictions.

Procedure

Jonathan Swift's Gulliver's Travels includes some ratios that the Lilliputians used to estimate proportions in order to construct clothes for Gulliver.

> "They measured my right thumb, and desired no more; for by a mathematical computa tion, that twice round the thumb is once round the wrist; and so on to the neck and the waist ..."

Part 1:

Have students work in groups of 3 or 4. Using a measuring tape they are to measure the thumb, wrist, and neck of each member of the group. Record the data on the group measurement sheet (**B-107**).

Set up an appropriate coordinate system on the board and using the class data make a scatterplot using either (wrist, neck) or (thumb, wrist) measurements. (3/4 inch round sticky labels work well for this.)

Discuss the ratio used by the Lilliputians and explore the linear function $\mathbf{y = 2x}$. Graph this on the coordinate system and compare to the class data.

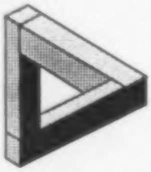
Part 2:

Using a graphics calculator enter the class data, perform a scatterplot of the data, and graph the line $\mathbf{y = 2x}$ on the calculator.

Part 3:

After discussing the previous two parts of the activity, the students are to write an explanation describing why they think the clothes the Lilliputians made for Gulliver fit just right, too loose, or too tight.

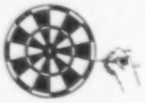
How Do They Fit?



Concepts and Competencies

Focus and Review: Solve equations, solve inequalities, and multiply binomials.

The puzzle format used in this activity can also be used to address: evaluating algebraic expressions, simplifying real number expressions, using algebraic properties, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, operating with polynomials, and solving quadratic equations.

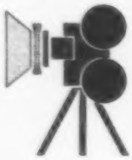


Goal

Use a puzzle to practice multiplication of binomials, solving simple linear equations and inequalities, and problem solving skills.

Materials

Copies of sample puzzle (**B-108, 109, 110, 111** separated) , transparency of sample puzzle (separated), puzzle master sheets (**B-112**).



Description

Students will assemble a 3 • 3 array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. **Well constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter.**

Procedure

Within a specific time limit (10-15 minutes), pass out to each student a separated puzzle and have them assemble it. Be sure the students understand that adjacent sides of puzzle pieces must match mathematically in order to solve the puzzle.

As students complete their puzzle, they raise a hand, the teacher checks and, if correct, gives a reward (bonus points, homework pass, treat, etc.).

After time is up, have the students direct the teacher in the assembly of the transparency version of the puzzle at the overhead. If time is short, display a transparent version of the uncut puzzle.

As an assignment, give each student two blank puzzle forms (**B-112**) so that they can create their own puzzle.

A student would first fill in the sheet so that adjacent sides matched up.

Scramble the pieces and copy the scrambled version on the second sheet.

After the new puzzles have been scored, edited, and cleaned up (neatly rewritten or typed), they are ready to be reproduced and separated (store puzzles individually in envelopes). The teacher now has a large pool of puzzles to use occasionally and allows several students to each be working different puzzles.

Use a 4 • 4 puzzle form to create additional puzzles.

The Wave

Concepts and Competencies

Focus:	Determine if data is behaving in a linear fashion, write the equation that models the data, and use the model to make predictions.
Review:	Graph ordered pairs and interpret information related to the points.

Goal

The students will collect and interpret data, determine a linear function of the time to complete the "wave" dependent upon the number of students participating, and use the linear function to make predictions.

Materials

Stopwatch, graph paper (for each student).

Description

In a whole class setting, increasingly larger groups of students (for example, 5 the first time, 10 the next, 15 the next) will perform the "wave". The number of students and the elapsed time for each group should be recorded and later graphed. Independent (participants) and dependent (time) quantities should be discussed and identified by the students. Students need to discuss any patterns exhibited in their data. A linear function should be determined from the data and predictions made using that function.

Procedure

Arrange the students around the classroom and practice first making waves with the whole class. To make a wave from a standing position, have the students put their hands on their knees. When the timer says "Go" have the students make a wave. The first student raises their hands above their head and then returns them to their knees, the second does the same, and so on. The last student says "Stop" as he or she finishes.

Begin first with 5 students and have the timer record the elapsed time.

Repeat the wave with 10 students and again with 15 students.

Based on the data collected so far, have the students estimate how long it will take 20 students to complete a wave.

Then have 20 students perform the wave and report the elapsed time.

Discuss with students independent and dependent quantities. That is, does the time depend upon the number of participating students or do the number of participating students depend upon the time? Generally speaking, **x** represents the independent quantity and **y** the dependent quantity.

Plot the points on the overhead projector as the students (you may choose to have students work in pairs) do the same at their desk.

Using a straight edge, have the students draw a line through two of the points so that the line appears to fit all of the points best.

Have the students extend their lines so that they can estimate how long it would take the whole class (for example, 27 students) to perform the wave. Check the estimate by having all the students perform the wave.

Have each student (or pair of students), using the points they chose earlier, find the slope of their line and its y-intercept. Students should define the slope and intercept in the context of the activity.

Give students the opportunity to determine a best-fit linear equation using a graphing calculator or appropriate computer software and compare it with their earlier effort.

How Do You Measure Up?

Concepts and Competencies

Focus:	Determine if data are behaving in a linear fashion.
Review:	Graph ordered pairs on the coordinate plane and interpret information related to the set of points.

Goal

To help students collect data, organize data, interpret results, and make predictions.

Materials

Measuring tape, graphing calculators, graph paper.

Description

Students will measure their forearm and height. The collected data will be organized, interpreted, and used to make predictions.

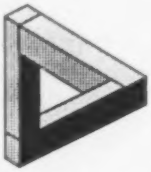
Procedure

Divide students into pairs. Students will measure and record the forearm length (distance from elbow to wrist) and height of their partners. When all the measurements are made, a composite list will be made for the class.

Students will input all data into their graphics calculator and display a scatter plot . From the scatter plot, students will determine if the data forms a linear pattern. Use the appropriate utility on the calculator to find an equation for a line-of-best-fit. Use the equation to make predictions about other measurements.

As an option, students may be expected to graph the data collected and line-of-best-fit on graph paper.

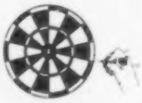
What Shape Are You?



Concepts and Competencies

Focus: Gather data, determine if the data is behaving in a linear fashion, compare the data with the **y = x** line, and interpret the results.

Review: Graph ordered pairs on the coordinate plane and interpret information related to the set of points.

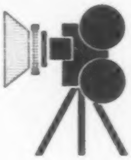


Goal

Students will graph their height vs arm span to investigate data that is above or below the line y=x.

Materials

Graph paper (**B-22**), a clear transparency for each group, measuring tapes, **B-115** transparency.

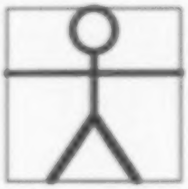


Description

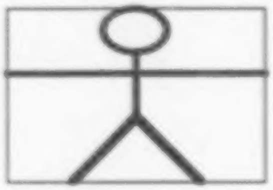
In groups, students measure their height and arm span and then graph the ordered pairs on a clear transparency laid over the graph sheet. You can overlay all of the transparencies on a graph sheet on the overhead to display all of the class' data. Students can then calculate the percent of students who are square, short-rectangular, or tall-rectangular.

Procedure

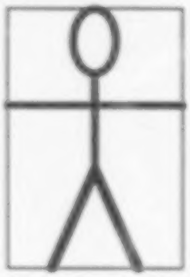
Ask students, "Are you square, short-rectangular, or tall-rectangular?" Show **B-115** .



Square
height = arm span



Short Rectangular
height < arm span

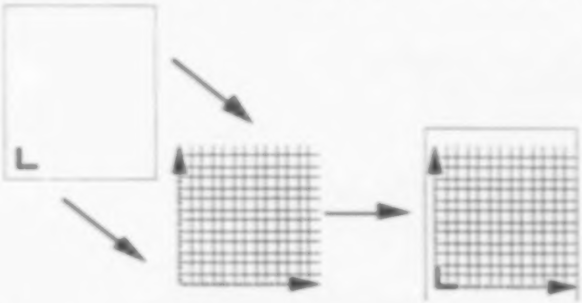


Tall Rectangular
height > arm span

Divide students into groups to measure the height and arm span (from fingertip to fingertip) for each person. Have each student write the ordered pair (arm span, height) for his measurements.

Show the overhead of the graph labeled arm span and height (use **B-22**). Discuss with the class where data would lie for people who are square. Have students write an equation to represent the set of data that are squares. (Let **x** = arm span, **y** = height) Graph this line on the transparency of **B-22**. Ask students if they are above or below the line **y = x**.

Prepare a transparency sheet for each group with a "corner" marked. Tell students to align this corner with the corner that is the intersection of the x- and y-axes on the graph sheet. Each student graphs his/her ordered pair on the transparency. Have each group write a description of how to decide from only the coordinates (without graphing) whether the ordered pair is above or below **y = x**.



Combine group results by overlaying the transparencies on the overhead. Have students calculate the percent of the class that is square, short-rectangular, and long-rectangular.
Have each group predict the total number of squares, short-rectangles, and tall-rectangles in the school.

It's All Downhill From Here

Concepts and Competencies

Focus:	Write and use the linear equation that models a set of data.
Review:	Graph ordered pairs in the coordinate plane and interpret information related to the set of points.

Goal

Create a linear model to represent rolling a ball down a ramp. Use the model to predict how far the ball will roll down a ramp of given height.

Materials

Measuring tape, golf balls (or some other object to roll down the ramp), ramp construction materials (paper towel tubes, meter sticks, boards, etc. for the balls to roll down and something that will uniformly raise the ramp - books, bricks, etc.), graphing calculators, recording sheets (**B-116**).

Description

This activity will have students building ramps and recording the distance a ball will roll down the ramp. Data will be collected from ramps of various heights. The data will be plotted on a scatterplot and line fitting techniques will be employed to produce a model. The function of height versus distance rolled produces a linear model which students will use to make predictions on distances referencing ramp heights from the model.

Procedure

Part 1:

Have students work in groups of 3 or 4. Students will need to disperse around the room to have enough space for rolling the ball across the floor.

Provide materials for building ramps and have students create ramps of 3 different heights.

For each height conduct 3 trials (roll the ball down the ramp and record the distance).

Part 2:

Input the data into the graphics calculator and view the data in a scatterplot.

Perform a regression analysis and create an equation that models the data.

Graph the equation that models the data.

Part 3:

Use the equation model from Part 2 to predict the distance traveled if the ramp were 4 units (bricks, books, etc.) tall.

Create a ramp 4 units tall and compare the actual results to the predicted results.

Part 4:

Write an explanation of how accurate your model is and possible reasons for error.

Adapted from "Roller Ball", AIMS Education Foundation.

Connecting Units of Measure

Concepts and Competencies

Focus: Determine if data is behaving in a linear fashion and interpret the slope of a linear model.

Review: Graph ordered pairs and interpret information related to the points.

Goal

Create a graph of the relationship between centimeters and inches based on several measurements and discover its linear nature.

Materials

Measuring tapes (or rulers), graphing calculators, recording sheets (**B-117**).

Description

Students will measure several objects in the classroom using both centimeters and inches. Students will plot corresponding pairs of measurements on a graph and interpret the information to determine the relationship between centimeters and inches.

Procedure

Measure the dimensions of several objects in the classroom using both centimeters and inches. Record the measures on the data sheet (**B-117**).

Plot a graph of the measurements in centimeters (on the vertical axis) verses the inches (on the horizontal axis). Students should carefully study the range of measurements so that appropriate scales can be chosen for the x- and y-axes.

Draw a best-fit line for the points plotted. Choose two points on or close to the best-fit line, naming them A and B.

Find the difference of the centimeter values and the inch values for points A and B. Record the calculations.

Find the slope of the line by dividing the difference in centimeter values by the difference in the inch values. Record the calculations.

Find the percent error by subtracting the slope from 2.54 and dividing the result by 2.54. Record the results. Discuss error and how it relates to this activity.

Repeat the procedure using a graphics calculator and compare the slope of the best-fit line determined by the calculator with earlier calculations.

Toothpick Triangles

Concepts and Competencies

Focus:	Create, graph, and use linear equations.
Review:	Graph ordered pairs in the coordinate plane and interpret information related to the set of points.

Goal

Use toothpicks to visualize patterns and use the data generated by the pattern to generate a linear relation and make predictions.

Materials

Toothpicks, graph paper, graphing calculators, recording sheets (**B-118**)

Description

This activity will have students building sets of triangles with toothpicks. Students will record data, plot the data, fit a line to the data, and make predictions based on the data. Students can complete the activity with only paper and pencil or calculators can be included to expedite the data analysis.

Procedure

Have students work in pairs and complete the worksheet (**B-118**) using toothpicks to create the necessary models.

Data collected should be plotted and fitted with a line.

Determine the slope of the line and relate it to the data gathered.

Write an equation to represent the data and use the equation to predict the number of toothpicks necessary to extend the pattern of triangles.

Estimating Fish Populations

Concepts and Competencies

Focus: Ratios and proportions.

Review: Draw inferences and construct arguments based on an analysis of data.

Goal

Students will use proportions to investigate the capture and recapture method for estimating a wildlife population.

Materials

Calculators, lima beans (or other consumable manipulatives)

Description

In the fall, county wildlife officials can approximate the number of fish in a pond in order to study the fish population and restocking needs. The method that they use is called the capture and recapture method. Students will use a simulation of this method and proportions to calculate a sample fish population.

Procedure

In estimating a fish population, officials catch a small sample of fish and place plastic tags on them that will not hurt them. The fish are then released back into the pond. Suppose 12 fish are tagged and returned to the pond. Several days later, the official returns and catches another quantity of fish, say 15. The number of fish tagged in the sample of 15 are counted. If 3 fish in the sample of 15 are tagged then a proportion can be used to estimate the total fish in the pond. Let n be the total number of fish in the lake:

$$\frac{12}{n} = \frac{3}{15} \qquad \frac{\text{no. of tagged fish}}{\text{total population}} = \frac{\text{no. tagged in sample}}{\text{total in sample}}$$

In order to obtain a more accurate estimate, samples of the same size are taken from different areas of the pond and the mean is found for the number of tagged fish.

No. of tagged fish	No. in Sample
1	15
4	15
2	15
1	15
2	15
4	15
2	15
0	15
1	15
3	15

The average number of fish tagged is 2.

$$\frac{12}{n} = \frac{2}{15}$$

$$2n = 180$$

$$n = 90$$

There are approximately 90 fish in the pond.

Discuss with the class how proportions can be used to find unknown quantities. For example, if in our class there are 3 left-handed students out of 30, how could we estimate the number of left handed students in the school ? Discuss how to refine the estimate by taking data from several classrooms. (An estimate of the percentage of left-handed people is 13%)

Divide class into groups. Give each group a shoe box with an unknown number of lima beans (or use two types of fish crackers). Have students model the capture and recapture method.

- Take out 12 beans, mark these with a red dot, and return them to the box. Shake the box.
- Select a sample of 15 beans, then record the number of marked beans that are in the sample. Return the beans to the box.
- Continue to take ten samples.

Have students use their data to calculate a proportion to estimate the fish population. Students can compare their estimate with the actual number of beans in the box.

Basketball: With the game on the line ...

Concepts and Competencies

Focus: Work with ratios, proportions and percents.

Review: Perform operations with real numbers.

Goal

On April 8, 1996, the Chicago Bulls and Charlotte Hornets of the NBA were playing. With 19 seconds left in the game and Chicago leading 97-96, Dell Curry of Charlotte was fouled. As Curry was preparing to shoot his two free throws, **how many points were the coaches expecting him to score?** For the season, Dell Curry made 146 out of 171 free throws.

Materials

Recording sheets (**B-30**), calculators, telephone book (optional).

Description

Students, working in pairs, are going to simulate the end-of-game situation 25 times and keep a record of their results. Students will use a random number generator (telephone book, calculator) to simulate shooting free throws. Students will use their experimental results to predict the outcome of the game (win, lose, or overtime). There are three extensions to this activity on **B-31, 34, 35.**

Procedure

Random numbers can be used easily to simulate the free throws. Express Curry's free throw shooting percent as a decimal. (146/171 = .8538) Give each team of students a page from an old telephone book and pick a column. Let the last four digits of a phone number represent a four digit decimal fraction. For instance, let -1352 become .1352. (Also see **Calculator Tips, W-39**)

Which numbers represent a MADE free throw? a MISSED free throw?
One way to illustrate numbers which represent MADE or MISSED shots is to use a number line.

Which part of the number line represents a MADE free throw? MISSED free throw?
If the number is less than or equal to .8538 a free throw is MADE. If the number is greater than .8538, a free throw is MISSED.

What are the possible outcomes of shooting 2 free throws? (0, 1, or 2 points are possible.)

How can you score 0 points? 1 point? 2 points?

- 0 points are scored when 2 consecutive numbers are more than .8538 (2 MISSES).
- 1 point is scored when the first number is greater than .8538 and the second is less than or equal to .8538 (1 MISS and 1 MADE) **OR** the first number is less than or equal to .8538 and the second is greater than .8538 (1 MADE and 1 MISS).
- 2 points are scored when both numbers are less than or equal to .8538 (2 made).

Have the students read two numbers at a time and record how many points were scored in each 2-shot free throw situation on their sheet. In the Frequency Table have the students record how often they scored 0, 1, and 2 points.

According to the experimental results, how many points should be scored?
Compute the total points scored, the average points scored, and the free throw shooting percentage (free throws made divided by free throws attempted).

According to the average points scored, how many points should be scored?
On the chalkboard (or on a transparency) record the Frequency Table results of all the teams. Total each category. Create a Class Frequency Table and find total points scored and average points scored for the class. Have each team compare the class results with their results. Individually, have the students write a paragraph indicating how many points a coach would expect to be scored in the original situation and use today's results to justify their answer.

The Month of Algebra

Concepts and Competencies

Focus: Students will perform operations with polynomials.

Review: Students will perform operations with real numbers.

Goal

Students will show algebraically why operating on a specific set of numbers will generate a particular pattern.

Materials

Calculators, calendar transparency, student copies of the calendar (**B-119**)

Description

In pairs, students will select several sets of dates and complete computations according to teacher directions. By replacing the numbers in a set with appropriate variable expressions and repeating the computations, students will be able to algebraically justify the pattern.

Procedure

Students select five consecutive dates. Add the first, second, fourth, and fifth numbers and divide the sum by 4. Try several other sets of five consecutive dates. What happens each time? Why does this occur each time?

Example: $(12 + 13 + 15 + 16) \div 4 = 14$

$[\, n + (n + 1) + (n + 3) + (n + 4) \,] \div 4 =$

$[\, 4n + 8 \,] \div 4 = n + 2$

Algebra				1	2	3
4	5	6	7	8	9	10
11	**12**	**13**	**14**	**15**	**16**	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Students select four consecutive dates. Find the product of the first and fourth numbers. Find the product of the second and third numbers. Compare the products. Try several other sets of four consecutive dates. What happens each time? Why does this occur each time?

Example: $19 \cdot 22 = 418,\ 20 \cdot 21 = 420$

$n(n + 3) = n^2 + 3n$

$(n + 1)(n + 2) = n^2 + 3n + 2$

Algebra				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	**19**	**20**	**21**	**22**	23	24
25	26	27	28	29	30	31

Students select three consecutive vertical dates. Find the product of the first and third numbers. Square the second number. Compare the results. Try several other sets of three consecutive dates. What happens each time? Why does this occur each time?

Example: $17 \cdot 31 = 527,\ 24^2 = 576$

$n(n + 14) = n^2 + 14n$

$(n + 7)^2 = n^2 + 14n + 49$

Algebra				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	**17**
18	19	20	21	22	23	**24**
25	26	27	28	29	30	**31**

Students select a three-by-three array of dates. Find the two products of opposite corners of the array. Compare the products. Try several other three-by-three arrays. What happens each time? Why does this occur each time?

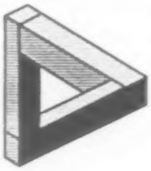
Example: $11 \cdot 27 = 297,\ 13 \cdot 25 = 325$

$n(n + 16) = n^2 + 16n$

$(n + 2)(n + 14) = n^2 + 16n + 28$

Algebra				1	2	3
4	5	6	7	8	9	10
11	**12**	**13**	14	15	16	17
18	**19**	**20**	21	22	23	24
25	**26**	**27**	28	29	30	31

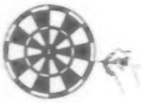
Multiplication with Algebra Tiles



Concepts and Competencies

Focus: Find the product of two binomials

Review: Use the associative and distributive properties.

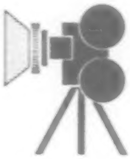


Goal

Use the rectangular area model of multiplication to multiply binomials and develop strategies for multiplication with algebraic expressions.

Materials

Algebra tiles (although a blackline master, **B-121**, is provided for you to create your own tiles, there are several "brands" of tiles commercially available with student and overhead versions), setup sheets (**B-120**), recording sheets (**B-122**).



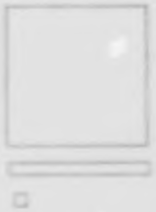
Description

Working in pairs, students will use pairs of binomials as the dimensions of a rectangle. The students will use the algebra tiles to build rectangles of given dimensions and find the area of the rectangle, the product of the binomials.

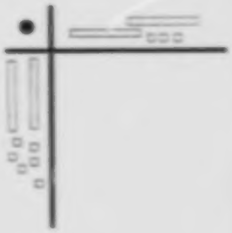
Procedure

In a **whole class** setting (at the overhead), talk with the students about the dimensions and areas of the three different tiles.

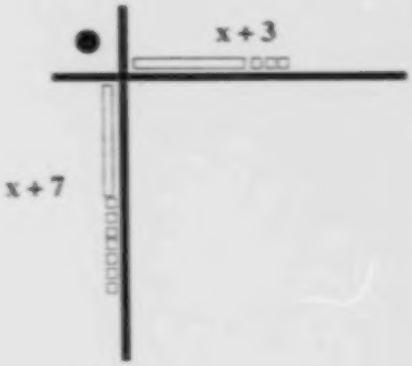
What is the shape of the large tile? The length of one side of this tile is **x** units. What is its area?
What is the shape of the long tile? The long side is **x** units long and the short side is one unit long. What is its area?
What is the shape of the small tile? The length of one side of this tile is one unit. What is its area?



With a transparency of the setup sheet, indicate to the students at the overhead that the two smaller tiles will be used as units of measure when building rectangles on the setup sheet.



The teacher at the overhead and the students at their desks, select the first problem, **(x + 3)(x + 7)**, from the recording sheet and lay the appropriate tiles end-to-end along the boundary of the "T".

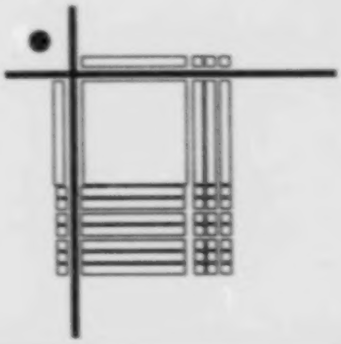


Use **all** the tiles to create a rectangle that has the dimensions **x + 7** and **x + 3**.

What is the area of the figure you have created? In other words, how many and which tiles did you use?

Complete the rest of the recording sheet.

Ask the students to discuss strategies for multiplying binomials without tiles.



Factoring Trinomials with Algebra Tiles

Concepts and Competencies

Focus : Factor a simple quadratic trinomial.

Review: Multiply binomials and use the distributive and associative properties.

Goal

Use the rectangular area model of multiplication to factor trinomials and develop strategies for factoring quadratic expressions.

Materials

Algebra tiles (although a blackline master, **B-121**, is provided for you to create your own tiles, there are several "brands" of tiles commercially available with student and overhead versions), setup sheets (**B-120**) , recording sheets (**B-123, 124**).

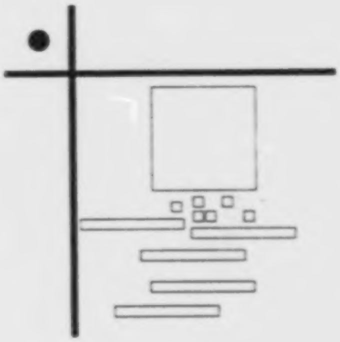
Description

Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

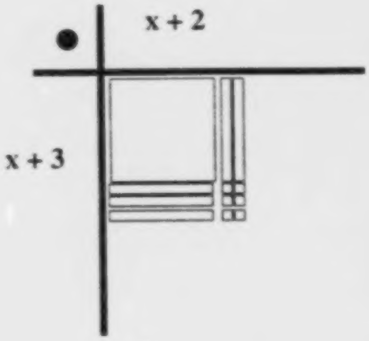
Procedure

In a **whole class** setting (at the overhead), talk with the students about the dimensions and areas of the three different tiles.

What is the shape of the large tile? The length of one side of this tile is **x** units. What is its area?
What is the shape of the long tile? The long side is **x** units long and the short side is one unit long. What is its area?
What is the shape of the small tile? The length of one side of this tile is one unit. What is its area?



With a transparency of the setup sheet, indicate to the students at the overhead that the tiles will be used as units of area measurement corresponding to the terms in the trinomial. They will be used to construct rectangles on the setup sheet.



The teacher at the overhead and the students at their desks select the first problem, $x^2 + 5x + 6$, from the recording sheet and arrange the appropriate tiles in the shape of a rectangle (no "holes" allowed).

What are the dimensions of the rectangle?

Complete the rest of the recording sheet.

Ask the students to discuss strategies for factoring trinomials without tiles.

Operating With Binomials

Concepts and Competencies

Focus:	Perform operations with polynomials and use an automatic grapher to relate the solutions of quadratic equations and the x-intercepts.
Review:	Solve a linear equation graphically.

Goal

Students will use the graphing calculator to solve quadratic equations by making connections among the linear factors of a quadratic expression, the solutions of the equation counterparts, and the x-intercepts of the equations.

Materials

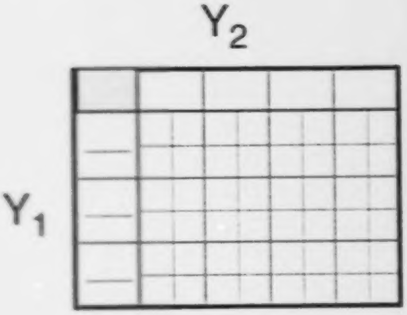
Graphing calculators, reporting sheets (**B-125, 126, 127**).

Description

Students will fill in the entries for $\mathbf{Y_1}$ and $\mathbf{Y_2}$ with binomials and, using the calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.

Procedure

Students can work individually or in pairs.

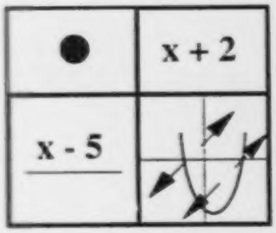


Give each student reporting sheets (**B-125, 126**).

Teachers should work along with the class on an overhead calculator to demonstrate the graphing routine for the first pair of binomials.

On the calcultor, enter **x - 5** for Y_1 and **x + 2** for Y_2.

Graph and record in the appropriate space on the reporting sheet.



Identify the x-intercepts for Y_1 and Y_2.

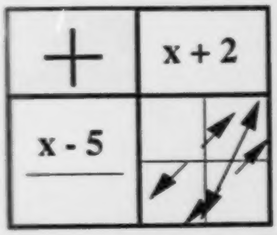
On the calculator, enter $Y_1 \bullet Y_2$ [or (x - 5)(x + 2)] for Y_3.

Graph and record in the same space where Y_1 and Y_2 were recorded.

Identify the x-intercept(s) for Y_3 and compare with those for Y_1 and Y_2.

Additional entries for Y_1 (**x + 3** and **2x + 4**) and Y_2 (**x + 5**, **x + 3**, and **3x - 7**) are provided.

Students should complete the table and answer the questions on the second report sheet.



Using the blank reporting sheet, teachers can make their own selections for Y_1 and Y_2 and change operations.

Getting to the Root of the Number

Concepts and Competencies

Focus: Students will find approximations for square roots.

Review: Students will perform operations with real numbers.

Goal

Students will be able to find rational numbers that approximate square roots.

Materials

Base-10 blocks, transparency of square numbers (**B-128**), transparency of example (**B-129**), recording sheets (**B-130**), calculators.

Description

Working in pairs, students will use base-10 blocks to build incomplete squares that represent a specified number. The square root of that number is the sum of the length of a side of the last completed square and the fraction whose numerator is the number of extra units and denominator is the number of units necessary to complete the next larger square. Students should compare the results with the decimal approximations generated by their calculators.

Procedure

Define square root as the length of the side of a square.

Use the transparency of the squares (**B-128**).

> What is the area of A? (100) How long is a side of A? (10)
> The square root of 100 is 10.
> Discuss square B in a similar fashion.

Individually or in pairs, ask the students to build, with the base-10 blocks, a twelve by twelve square and an eleven by eleven square. Show C and D from the transparency and discuss square roots in that context.

Use transparency **B-129** ($\sqrt{}$ 135)

> Select the blocks necessary to build a square of 135 units (step A). Ask the students to build a square using all the units.
>
> What is the largest square that can be built? (11 • 11, step B)
>
> How many more units are needed to build the next larger square? (23, for a 12 • 12)
> How many do you have for that task? (14, steps B and C)

Since 135 is between the square numbers 121 and 144, then the square root of 135 is between 11 and 12. Since we have 14 of the 23 units necessary to create the 12 • 12 square from the 11 • 11 square, we can say that the square root of 135 is 11 and 14/23.

> Compare 11 and 14/23 with $\sqrt{}$ 135 on the calculator. (11and 14/23 ≈ 11.609, $\sqrt{}$ 135 ≈ 11.619)

Use the blocks to determine rational approximations for the square roots of other numbers (**B-130**)

Quadratic Functions

Concepts and Competencies

Focus: Use a calculator to find the solution(s) to a quadratic equation and relate the solution(s) to the x-intercept(s).

Review: Simplify radicals and approximate square roots.

Goal

Students will find solutions to quadratic equations using two different methods, the quadratic formula and graphing to find the x-intercepts.

Materials

Graphing calculators, recording sheets **B-131, 133**.

Description

Students will use the quadratic formula to obtain a solution, in radical form, to the related quadratic equation. Using a calculator, they will convert the solutions to decimals, rounding to tenths. Students will then graph the quadratic function on a graphics calculator. They will use the zoom and trace keys to estimate the x-intercepts of the function to the nearest tenth (or they may use the table function).

Procedure

Part I

Discuss with students how to write a quadratic equation from the quadratic function.

Have students solve the equation using the quadratic formula. Students should record the solutions on their recording sheet (**B-131**) in both radical and decimal (nearest tenth) form.

Students can then graph the function using a graphics calculator. Sketch the graph on the recording sheet in the space provided. They can use zoom and trace keys to locate each x-intercept. Record the intercepts, rounding to the nearest tenth.

Discuss with students how they are using two different methods to find the values for x in a quadratic equation that is equal to 0.

The graph gives the students a visual representation of the concept while the quadratic formula allows students to use calculations.

Part II

Have students write their explanations to the discussion questions (**B-133**).

For further exploration, have students try the bonus!

Open Boxes

Concepts and Competencies

Focus:	Use the calculator to explore the graphs of functions and interpret a graph in a real-world setting.
Review:	Graph ordered pairs of numbers on the coordinate plane and interpret information related to the set of points. Multiply polynomials.

Goal

Collect and analyze data to determine the maximum value for a function which models the volume of a three dimensional figure.

Materials

Scissors, rulers, tape, grid paper (**B-21**), recording sheets (**B-134**), graphing calculators.

Description

In groups, students will collect and analyze data from the construction of several boxes and estimate the maximum volume. Students will use an algebraic model to determine a maximum volume and compare it with the experimental results.

Procedure

In a **whole class** setting, demonstrate to students how to make an open box from the grid paper by cutting out square corners, folding up the sides, and taping the corners. For example, cut out the 16 by 23 centimeter grid and cut out 3 by 3 cm squares from each corner. Fold up the resulting sides and tape on the corners. An open box will result.

Discuss these questions with the students.

- Will cutting out different size corners result in different sizes of open boxes?
- And if so, which box will hold the most?

Arrange students in groups. Give each group 10 sheets of grid paper, scissors, and tape.

Ask the students to create the boxes that result from cutting out corners that are 1 by 1, 2 by 2, ... , and 10 by 10 centimeters.

Post the boxes where the entire class can view them.

Ask the students to vote on the box which they think will hold the most based only on visual appearances.

Each group should record the dimensions of their boxes and calculate the volumes.

Have the students discuss and record which box has the largest volume and compare with their earlier prediction.

Using their calculators, students should graph height verses volume on their calculators and identify the maximum volume on the graph.

Discuss with students how to determine the algebraic expression that represents the volume of **all** the boxes. Use **B-135** and **B-136** when appropriate.

Have the students graph the equation $y = x(16 - 2x)(23 - 2x)$.

Use the calculator to determine the maximum volume and the corresponding dimensions.

Ask the students to describe the location of the maximum volume on the graph.

Discuss with the class the parts of the graph of $y = x(16 - 2x)(23 - 2x)$ that are not relevant to the discussion of the problem. ($x < 0$ **and** $x > 8$)

The Maximum Garden

Concepts and Competencies

Focus: Maximum values of quadratic functions

Review: Formulas, functions, area

Goal

Students will be able to apply problem solving skills to design a garden with maximum area.

Materials

Recording sheets (**B-137, 138, 139**), computer spreadsheet or graphing calculator.

Description

In this activity, students will use a table to list possible values for the dimensions and area of a garden space. Students will write an equation to graph the width of the garden versus the area. This will be a quadratic function with a maximum value. See **B-140** for extensions.

Procedure

Discuss with the class the vegetable garden problem.

Begin with the 1.2 meter fence sections and have students complete the table using a spreadsheet or calculator.

On a spreadsheet, students can enter the values for the possible widths, then use formulas to calculate the length and area with the "fill down".

The students can either hand graph the width versus the area or draw the graph using the spreadsheet charting capabilities. Example:

	A	B	C
1	1.2	= 18-2(A1)	= A1*B1
2	2.4		
3	3.6		
4	4.8		

Have students complete in the same manner, the table using fence lengths of .9 meters. Students can discuss which fence length they would want to use and why.

With the little difference in the maximum area, the price might be a determiner or the time to install the smaller sections vs the larger sections.

To complete using the TI-82 table generator, enter the following into the table setup:

2nd TBLSET

TABLE SETUP
TblMin = 0
Tbl = 1.2
Indpent: Auto Ask
Depend: Auto Ask

Enter the equations **y = 18 - 2x** into the **Y=** menu to calculate the length and **y = x (18-2x)** to the calculate the area. Students can view the table to obtain the width and area for the increments of 1.2.

Have students enter the values from the table on the recording sheet and hand graph the width verses the area. Students can visually see the maximum area from the graph.

Students can change the table setup for increments of .9 to complete the chart for the maximum area for the .9 m sections.

Shuttle Launch

Concepts and Competencies

Focus: Use quadratic equations to solve problems.

Review: Graph quadratic equations.

Goal

Using quadratic equations, students will identify the vertex, x-intercept, and several other points in order to report the performance of the space shuttle rocket booster.

Materials

Graphing calculators, recording sheets (**B-141**).

Description

Working in pairs, students will use a pair of quadratic equations to identify the critical points along the flight of the solid rocket boosters (SRB) that are used to launch the space shuttle. Students will connect algebraic ideas (intersection, vertex, x-intercept, and evaluating expressions) with points along the flight path (engine shutdown, maximum altitude, splashdown, and altitude versus elapsed time).

Procedure

The space shuttle uses solid rocket boosters (SRB) during the launch phase of its flight from Cape Canaveral. The SRBs burn for about two minutes, shut down, detach from the main rocket assembly, and fall back to Earth 140 miles downrange from the launch site. Parachutes assist the ocean landing beginning from an altitude of 20,000 feet. The SRBs are recovered and used again for a later launch.

Discuss a rocket launch with the students and, with the assistance of the media center, view a video of a space shuttle launch. Make sure the students pay particular attention to the two SRBs.

Pass out to each student a recording sheet (**B-141**). Ask the students to draw a graph illustrating the length of time the SRBs are in the air versus their altitude.

The students will use the following system of quadratic equations and descriptions to answer questions and report findings related to the flight of the SRBs. (x = elapsed time in seconds, y = altitude in miles)

When the SRBs are on and the shuttle is climbing: $\mathbf{y = 0.002x^2}$

When the SRBs are off and
detached from the main assembly: $\mathbf{y = -33 + 0.72x - 0.00176x^2}$

Graph the system of equations on the calculator and sketch on the recording sheet.

To the nearest second, when did the SRBs shut down?

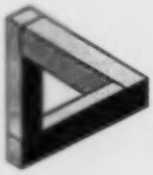
Determine the maximum altitude attained by the SRBs and at what time did that occur.

How long were the SRBs in the air?

Draw a more detailed graph (time vs. altitude) of the SRBs' flight and identify the critical points (launch, shut down and separation, maximum altitude, and splashdown).

Identify algebraically the critical points of the flight.

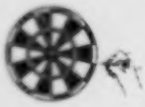
Patterns With Exponential Equations



Concepts and Competencies

Focus: Use formulas and calculators to explore and solve problems involving exponentials.

Review: Simplifying expressions using exponents.

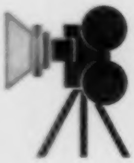


Goal

The students will investigate patterns when different values are used for the base, **b**, in an exponential equation, $\mathbf{y = b^x}$.

Materials

Recording sheets (**B-142, 143**), calculators.



Description

Students will graph equations in which the base **b** is a positive number greater than 1. They will investigate what happens as **b** increases and describe the pattern. Next, students will graph equations in which the base **b** is between 0 and 1 and describe this pattern.

Procedure

Many applications with exponential equations involve a percent increase or a percent decrease. Other applications may involve repeated multiplication such as doubling, tripling, or halving. This activity will allow students to investigate patterns in functions that are increasing and those that are decreasing.

Part I:
Have students make tables of values for **$y = 2^x$** and **$y = (1/2)^x$**. These may be calculated by hand to review exponents or on a calculator. If calculators have a table generator, this may be used. Then have students graph the two functions and discuss any differences.

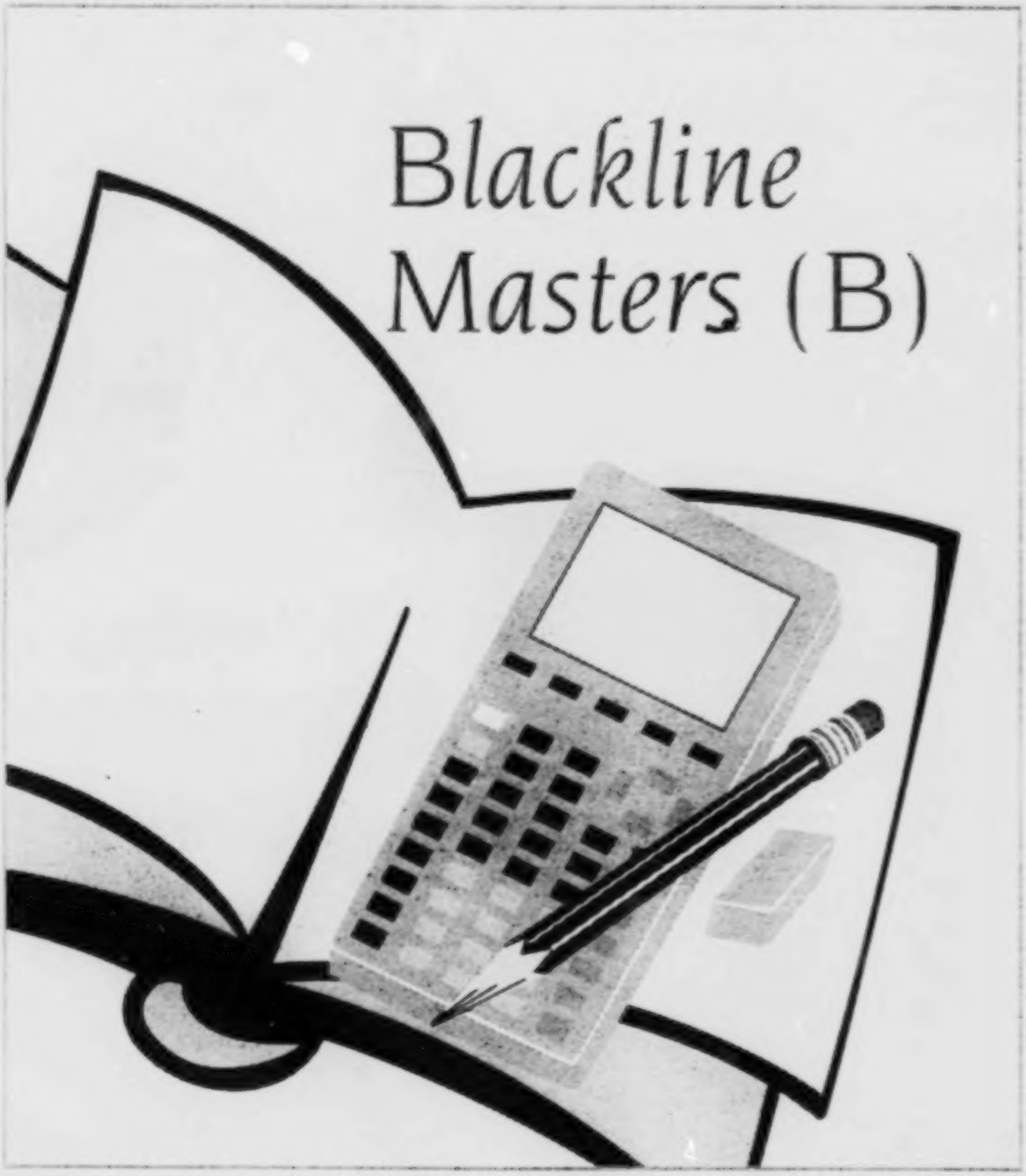
x	$y = 2^x$
-2	
-1	
0	
1	
2	

x	$y = (1/2)^x$
-2	
-1	
0	
1	
2	

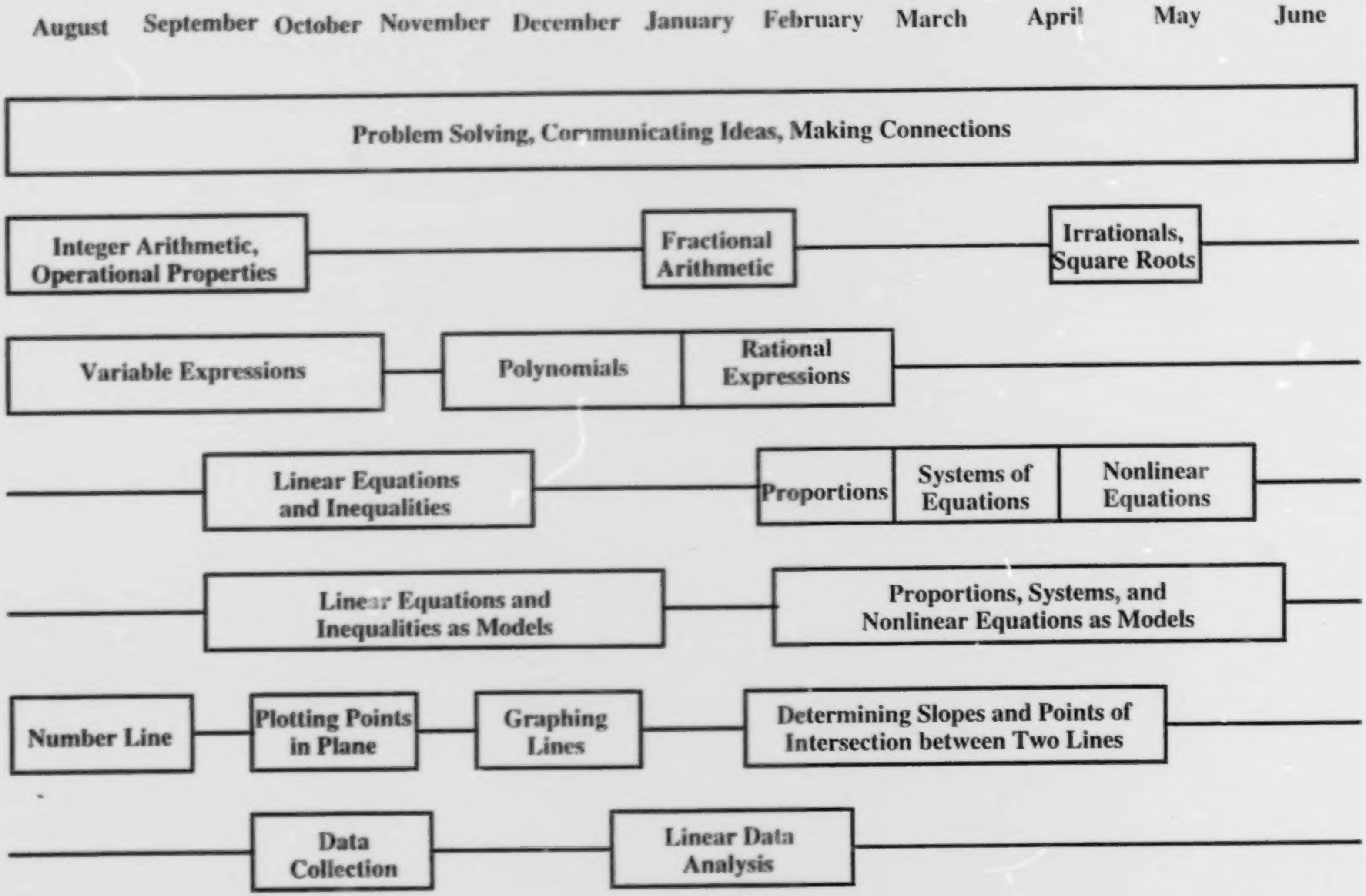
Part II:
Students can investigate patterns in exponential equations by graphing several using a graphing calculator and then sketching the graph on the recording sheet (**B-142**). Students also record the y-intercept and whether the graph is increasing or decreasing. Ask students to compare the first three graphs by graphing them on the same axis. Next, have them compare the last three graphs on the same axis.

Part III:
Discuss with class how the equations for a percent increase or decrease follow the formula **$T = A(1+r)^n$** where **T** = total amount, **A** = initial amount, **r** = % increase or decrease, and **n** = number of years.

Blackline Masters (B)



Instructional Pacing Guide Outline


August
September
October
November
December
January
February
March
April
May
June
Problem Solving, Communicating Ideas, Making Connections
Integer Arithmetic, Operational Properties
Fractional Arithmetic
Irrationals, Square Roots
Variable Expressions
Polynomials
Rational Expressions
Linear Equations and Inequalities
Proportions
Systems of Equations
Nonlinear Equations
Linear Equations and Inequalities as Models
Proportions, Systems, and Nonlinear Equations as Models
Number Line
Plotting Points in Plane
Graphing Lines
Determining Slopes and Points of Intersection between Two Lines
Data Collection
Linear Data Analysis

Instructional Pacing Guide Outline

Pacing by Objectives

August September October November December January February March April May June

Equations, Inequalities, and Applications

3.1 - 3.6 3.8 - 3.12 3.7 8.2 - 8.4 6.4 - 6.7 9.3 - 9.8

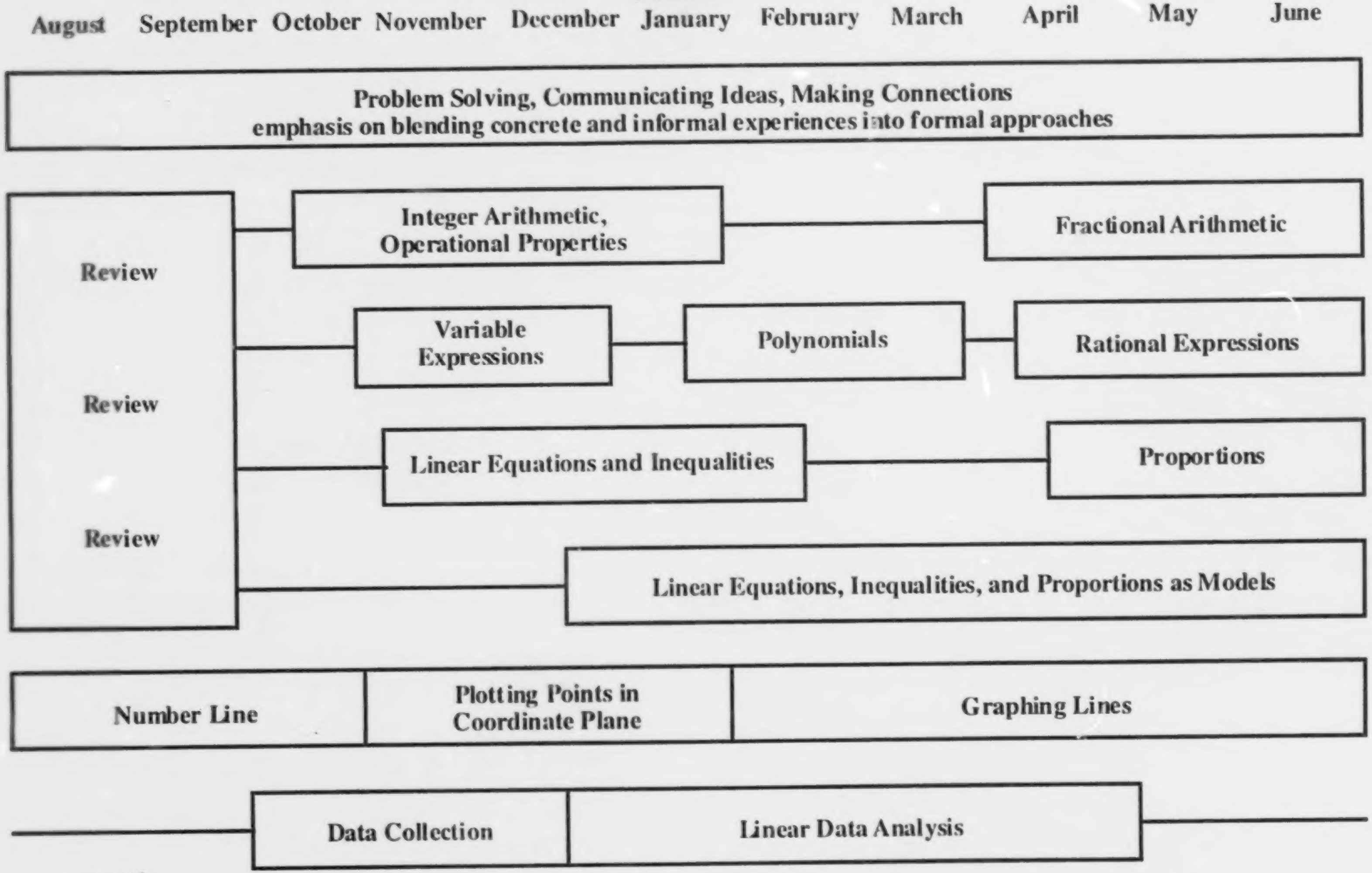
Number and Operation Sense; Using Symbols

1.1 - 1.4 2.1 - 2.5 7.1 - 7.11 8.1 2.6 - 2.10

Coordinate Geometry and Applications

4.1 - 4.3 4.4 - 4.10 3.3 5.1 - 5.9 5.10 6.1 - 6.3 5.11 9.1 - 9.6 9.9 - 9.10

Instructional Pacing Guide Outline
Algebra I, Two Year Sequence
Year 1



August September October November December January February March April May June

Problem Solving, Communicating Ideas, Making Connections
emphasis on blending concrete and informal experiences into formal approaches

Review
Review
Review

Integer Arithmetic, Operational Properties — Fractional Arithmetic

Variable Expressions — Polynomials — Rational Expressions

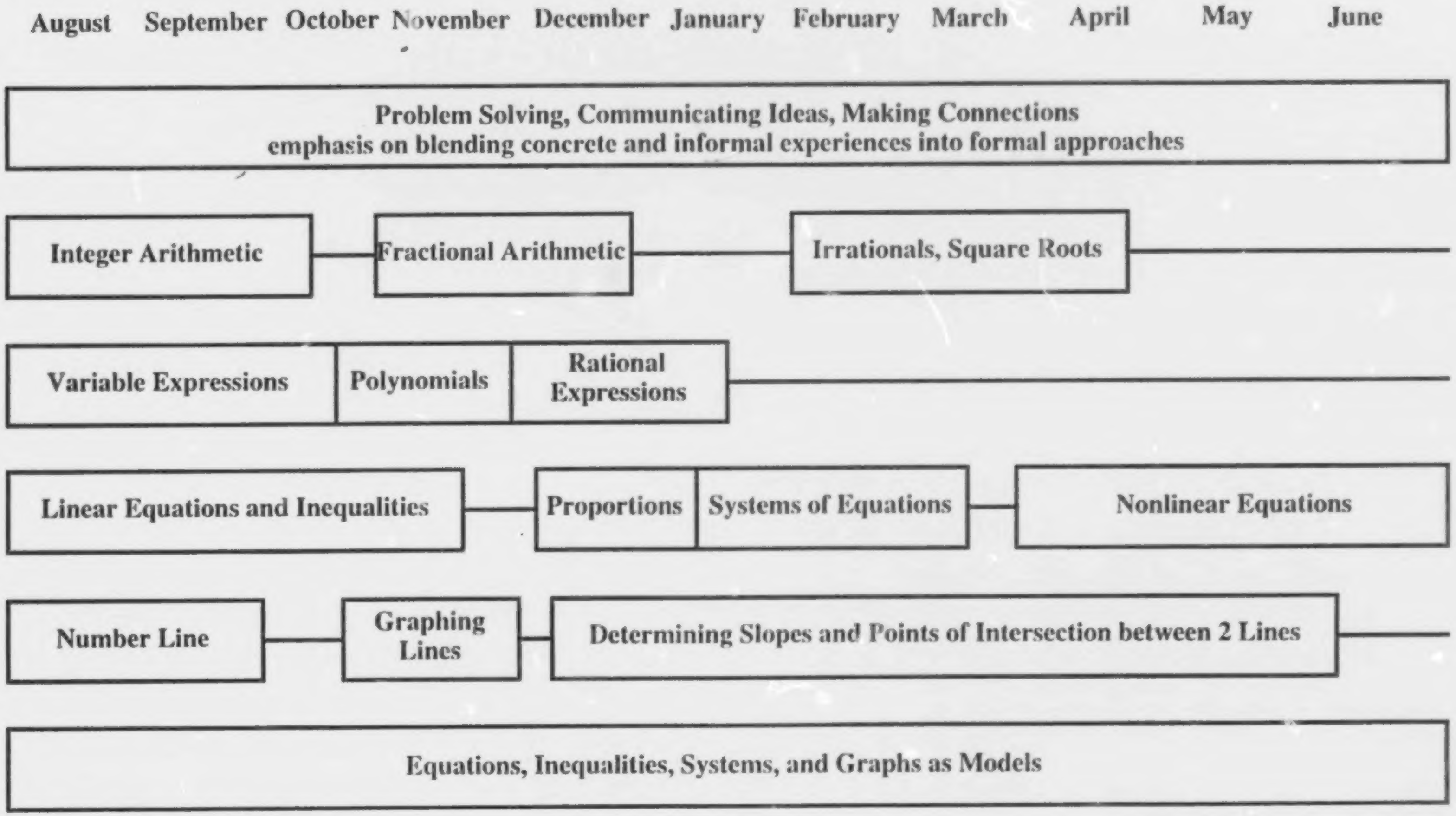
Linear Equations and Inequalities — Proportions

Linear Equations, Inequalities, and Proportions as Models

Number Line | Plotting Points in Coordinate Plane | Graphing Lines

Data Collection | Linear Data Analysis

Instructional Pacing Guide Outline
Algebra I, Two Year Sequence
Year 2


August
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Problem Solving, Communicating Ideas, Making Connections
emphasis on blending concrete and informal experiences into formal approaches
Integer Arithmetic
Fractional Arithmetic
Irrationals, Square Roots
Variable Expressions
Polynomials
Rational Expressions
Linear Equations and Inequalities
Proportions
Systems of Equations
Nonlinear Equations
Number Line
Graphing Lines
Determining Slopes and Points of Intersection between 2 Lines
Equations, Inequalities, Systems, and Graphs as Models

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will use the language of algebra. 1.1 Evaluate algebraic expressions 1.2 Use formulas to solve problems. 1.3 Translate word phrases and sentences into expressions and equations and vice versa. 1.4 Use the associative, commutative, and distributive properties.		
The learner will perform operations with real numbers. 2.1 Simplify real number expressions with and without a calculator 2.2 Determine the additive or multiplicative inverse of a number. 2.3 Determine the absolute value of expressions. 2.4 Raise a real number to an indicated power. 2.5 Write numbers in scientific notation and use this notation with the calculator. 2.6 Distinguish between rational and irrational numbers. 2.7 Find approximations for square roots with and without a calculator. 2.8 Simplify radical expressions. 2.9 Multiply two binomials which contain square roots. 2.10 Compare real number expressions.		

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will solve equations and inequalities with one variable.		
3.1 Solve a simple equation using the addition property of equality and the idea of additive inverse.		
3.2 Solve a simple equation using the multiplicative property of equality and the idea of multiplicative inverse.		
3.3 Solve an equation graphically and by using more than one property of equality.		
3.4 Solve an equation which contains similar terms.		
3.5 Solve an equation which has the variable in both members.		
3.6 Solve an equation in which the numerical coefficient is a fraction.		
3.7 Solve a formula for one of its variables or find the value of a variable when values of the other variables are given.		
3.8 Use problem solving skills to solve real world and "word" problems which involve a linear equation or a formula.		
3.9 Solve a simple equation involving absolute value.		
3.10 Solve a simple equation containing a radical.		
3.11 Find the solution set for a linear inequality when replacement values are given for the variables.		
3.12 Solve a linear inequality by using transformations.		
3.13 Use inequalities to solve problems.		
3.14 Find the solution set of combined inequalities.		

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will demonstrate an elementary understanding of relations and functions. 4.1 Graph and locate sets of real numbers on the number line. 4.2 Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points. 4.3 Find the distance between two points on a number line. 4.4 Graph a relation on the coordinate plane. 4.5 Distinguish between a relation and a function. 4.6 Graph a relation given an equation and a domain. 4.7 Sketch a reasonable graph for a given relationship. 4.8 Interpret a graph in a real world setting. 4.9 Use a computer or graphing calculator to explore the graphs of functions. 4.10 Compare ordered pairs to the line y = x and interpret the results.		

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will graph and use linear equations and inequalities. 5.1 Determine if data are behaving in a linear fashion. 5.2 Find the solution set of open sentences in two variables when given replacement sets for the variables. 5.3 Graph a linear equation in two variables. 5.4 Graph a line given its slope and y-intercept. 5.5 Find the slope of a non-vertical line given the graph of the line or an equation of the line or two points on the line. 5.6 Describe the slope in a real-world linear relationship using real-world terms. 5.7 Write the slope intercept form of an equation of a line. 5.8 Write the equation of a line given the slope and one point on the line, or two points on the line. 5.9 Write the equation of a line which models a set of real data. 5.10 Use the line which models real data to make predictions. 5.11 Graph a linear inequality in two variables.		

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will graph and solve systems of linear equations and inequalities. 6.1 Use a graph to find the solution of a pair of linear equations in two variables. 6.2 Graph the solution set of a system of linear in equalities in two variables. 6.3 Use a computer or graphics calculator to solve systems of linear equations. 6.4 Use the substitution method to find the solution of a pair of linear equations in two variables. 6.5 Use the addition or subtraction method to find the solution of a pair of linear equations in two variables. 6.6 Use multiplication with the addition or subtraction method to solve systems of linear equations. 6.7 Use systems of linear equations to solve problems.		

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Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will perform operations with polynomials. 7.1 Add and subtract polynomials. 7.2 Multiply monomials. 7.3 Find an indicated power of a monomial. 7.4 Multiply a polynomial by a monomial. 7.5 Find the product of two binomials. 7.6 Multiply two polynomials. 7.7 Divide two monomials. 7.8 Divide a polynomial by a monomial. 7.9 Find a common monomial factor in a polynomial. 7.10 Factor the difference of two squares. 7.11 Factor a simple quadratic trinomial.		
The learner will work with ratios, proportions, and percents. 8.1 Simplify ratios involving algebraic expressions. 8.2 Solve proportions. 8.3 Use ratios and proportions to solve problems. 8.4 Solve problems involving percents.		

Curriculum Resource Organizer

Goals and Objectives	Textbook	Other Resources
The learner will explore, graph, and interpret nonlinear equations. 9.1 Graph a quadratic equation. 9.2 Use an automatic grapher to find the solution to a quadratic equation. 9.3 Solve a quadratic equation when one member is in factored form and the other member is zero. 9.4 Solve a second degree equation by factoring. 9.5 Use an automatic grapher to relate the solutions of quadratic equations and the x-intercepts. 9.6 Understand that the vertex provides the maximum or minimum value of the function. 9.7 Solve a quadratic equation in which a perfect square equals a constant. 9.8 Solve a quadratic equation by using the quadratic formula. 9.9 Use quadratic equations to solve problems. 9.10 Determine if a set of data represents an exponential function. 9.11 Use formulas, calculators, and automatic graphers to explore and solve problems involving exponentials.		

Algebra in the NC K-8 Standard Course of Study (1989)

GRADE K
3.1 Describe likenesses and differences.
3.2 Sort by a given attribute; tell about classification.
3.3 Sort by own rule; explain rule.
3.4 Identify/ describe patterns.
3.5 Copy and continue simple patterns.
3.6 Create patterns with actions/ words/ objects.
3.7 Order familiar events; describe.

GRADE 1
3.1 Describe objects by their attributes; compare and order.
3.2 Sort by given attribute/ by more than one attribute; explain sorting rules.
3.3 Sort objects by own rule; explain sorting rule.
3.4 Copy/ continue patterns; translate into different forms.
3.5 Create patterns with actions/ words/ objects.
3.6 Find and correct errors in patterns.
3.7 Identify patterns in the environment.

GRADE 2
3.1 Compare/describe similarities and differences.
3.2 Classify by more than one attribute; describe rules used in sorting.
3.3 Define and continue patterns; translate into different forms.
3.4 Identify classification and patterning in the environment.
3.5 Identify/ correct errors in patterns.
3.6 Use patterns to continue numerical sequences.
3.7 Order objects and events; use ordinal numbers.

GRADE 3
3.1 Organize objects or ideas into groups; describe attributes of groups and rules for sorting.
3.2 Describe (demonstrate) patterns in skip counting and multiplication; continue sequences beyondmemorized/ modeled numbers.
3.3 Extend/ create geometric and numerical sequences; describe patterns.
3.4 Observe/ analyze patterns; describe pattern properties and give examples of similar patterns in varied forms.
3.5 Use patterns to make predictions and solve problems.
3.6 Use understanding of seriation in real life situations.
3.7 Explore number patterns with calculators.

GRADE 4
3.1 Identify and describe mathematical patterns and relationships that occur in the real world.
3.2 Demonstrate or describe patterns in geometry, data collection, and arithmetic operations.
3.3 Identify patterns as they occur in mathematical sequences.
3.4 Extend and make geometric patterns.
3.5 Given a table of number pairs, find a pattern and extend the table.
3.6 Use patterns to make predictions and solve problems; use calculators when appropriate.
3.7 Use intuitive methods, inverse operations, and other mathematical relationships to find solutions to open sentences.

Algebra in the NC K-8 Standard Course of Study (1989)

GRADE 5

3.1 Identify and describe patterns as they occur in numeration, computation, geometry, graphs and other applications.
3.2 Investigate patterns that occur when changing numerators and calculator investigations.
3.3 Use patterns to solve problems, make generalizations, and predict results.
3.4 Create a set of ordered pairs by using a given rule.
3.5 Given a group of ordered pairs, identify a rule to generate them or new pairs in the group, using calculators or computers where appropriate.
3.6 Model the concept of a variable using realistic situations.

GRADE 6

3.1 Represent number patterns in a variety of ways including the use of calculators and computers.
3.2 Use patterns to explore the rules for divisibility.
3.3 Use graphs and tables to represent relations of ordered pairs, using a calculator or a computer where appropriate; describe the relationships.
3.4 Identify and use patterning as a strategy to solve problems.
3.5 Use realistic examples or models to represent concepts and properties of variables, expressions, and equations. (Identity property of zero, Identity property of one.)
3.6 Use the order of operations to simplify numerical expressions, verifying the results with a calculator or computer.

GRADE 7

3.1 Describe, extend, analyze and create a wide variety of patterns to investigate relationships and solve problems.
3.2 Use concrete materials as models to develop the concept of operations with variables.
3.3 Use concrete, informal and formal methods to model and solve simple linear equations.
3.4 Investigate and evaluate algebraic expressions using mental calculations, pencil and paper and calculators where appropriate.
3.5 Given a simple equation, formulate a problem; solve and explain.

GRADE 8

3.1 Describe, extend, analyze and create a wide variety of geometric and numerical patterns, such as Pascal's triangle or the Fibonacci sequence.
3.2 Identify and define the commutative, associative and distributive properties; give examples and explain their meanings.
3.3 Analyze representations of data with tables, graphs, verbal rules and equations to explore the properties and relationships.
3.4 Using patterns and algebraic methods, solve problems, including those with integers.
3.5 Generate ordered pairs to graph a linear equation with and without a calculator.
3.6 Investigate non-linear equations and inequalities informally.

NCTM Curriculum Standards (1989): Algebra

Patterns and Relationships
In grades **K-4**, the mathematics curriculum should include the study of patterns and relationships so that students can

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

Patterns and Functions
In grades **5-8** the mathematics curriculum should include explorations of patterns and functions so that students can

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems.

Algebra
In grades **5-8**, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems.

Algebra
In grades **9-12**, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can

- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities;
- appreciate the power of mathematical abstraction and symbolism.

Suggested Classroom Accommodations for Students with Specific Learning Disabilities

Cognitive Strategies	Behavior	Accommodations
Remembering	forgets order of steps	chart of steps displayed
Self-managing	cannot explain concept	self-questioning taught
Information gathering	does not understand on first listening	frequent summaries paraphrasing strategy
Organizing	cannot make visual representation	vocabulary recorded with both words and a visualization strategy
Analyzing	cannot locate errors	verbal rehearsal strategy
Problem solving	cannot shift strategies	demonstrate each problem using two strategies
Time managing	poor assignment completion	prioritize assignments; required time chart for increased awareness of time demands
Integrating	poor notes	note taking strategy organized by concepts, not textbook chapters
Generating	weak concept connecting	prediction strategies pattern awareness
Evaluating	poor test taking	alternate tests; frequent assessment; test taking strategies

Some Additional Accommodations

- Modify original task to meet the needs of handicapped students.
- Provide taped material to listen to, rather than read.
- Emphasize higher use of objective test in contrast to subjective tests.
- Offer three choices instead of four in multiple-choice formats.
- Provide highlighted text for student use.
- Provide large print materials.
- Increase allowable time for completion.
- Reduce weight of test importance.
- Change fill-in-the-blank to multiple-choice format.

Creating "Real World" Problems

When hurricane Fran hit North Carolina on the evening of September 5, 1996, over one million homes and businesses were left without power. On September 14, *The News & Observer* of Raleigh displayed the following information in the form of a graph.

Date	Customers without power
Sept. 6	1,159,000
Sept. 7	804,000
Sept. 8	515,000
Sept. 9	340,500
Sept. 10	195,200
Sept. 11	136,300
Sept. 12	77,000
Sept. 13	37,600

Take advantage of the information provided and create problems related to topics you are presently studying. Use them for classwork, homework, or quiz/test items. Here are some possibilities.

- Make a scatterplot of the data and discuss any patterns or trends apparent. Estimate when all power would be restored. (4.2, 4.4, 4.8, 5.1, 9.10)

- Based on the initial number of customers without power, what percent of those customers had their power restored after one day? after two days? after three days? (2.10, 8.4)

- Using only the September 6 and 7 data, create a linear equation and use it to predict when all of the customers would have their power restored. (5.8, 5.9, 5.10)

- Find the equation of the best-fit curve (linear or exponential) and use it to predict when all of the customers would have their power restored. (5.1, 5.6, 5.10, 9.10, 9.11)

Dear Parents:

On **April** _____ we will be having a "Portfolio Review" session in the Mathematics Department of _____________ HS. Students in __________________ classes will be participating. It will be an opportunity for you to be able to see and evaluate some of your child's best work for this semester. This will count as one major test grade for this grading period. Therefore it is important that both you and the student attend. Your child will sit down with you and explain why they selected each item in their portfolio as an example of his or her best work.

You will be viewing a logo, a math button, a famous mathematician project, and a collection of their best tests, quizes, homework, and other items of the individual teachers choice. Please plan to join us on April _____ between 4 and 8 PM. The review will probably take between 15 and 30 minutes.

Please indicate the time you will be able to attend on **April** _____.

4- -4:30 _____	5 - 5:30 _____	6 - 6:30 _____	7 - 7:30 _____
4:30 - 5 _____	5:30 - 6 _____	6:30 - 7 _____	7:30 - 8 _____

Student signature ____________________ Parent signature ______________________

Please return by April _____. Thank you in advance.

Portfolio Evaluation Sheet

Thank you very much for attending our portfolio review session this evening. Please listen carefully as your student tells you something about each item that is included in his or her portfolio. Please place check (√) in the space provided to indicate that the student has each of the indicated items in his or her portfolio. Place a (√-) if the student has some but not all of the required items for that category. If the student has no items for a particular category, please leave the space blank.

Homework (8)	_____	(20 points)
Test or quizes (3)	_____	(20 points)
Quiz (1) (optional)	_____	(5 points)
A Math Logo	_____	(15 points)
A Math Button	_____	(5 points)
Mathematician Project	_____	(15 points)
Career poster	_____	(15 points)
Pi Day Project	_____	(10 points)

How well did your student complete and explain the content of the portfolio.

A	93 - 100	_____
B	85 - 92	_____
C	77 - 84	_____
D	70 - 76	_____
E	0 - 69	_____

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Mathematician Project

I. Select a mathematician from the list provided. The name of the mathematician that you select is due on ____________________.

Total points (5 max) ______________.

II. Take a large sheet of construction paper and fold it in half.

A. Design a neat and original (10 points) cover for your book. The following should be on your cover:

1.	Name of the mathematician	5 points
2.	Some aspect of their life or work	5 points
3.	Color	5 points
4.	Your name as author	5 points

Total points (30 max) ______________.

B. The first inside page should be in ink or typed (5 points) and have:

1.	Your name	5 points
2.	File number	5 points
3.	Class period	5 points
4.	Name of mathematician	5 points
5.	List of sources of information	5 points

Total points (30 max) ______________.

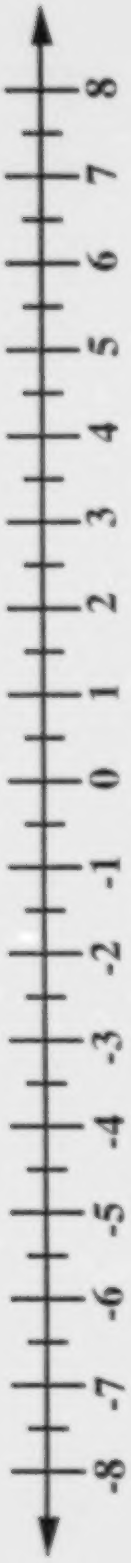
C. The other inside pages are the report on the mathematician which should be written in ink or typed (5 points) and should include the following:

1.	Personal information about the mathematician such as nationality and dates lived	10 points
2.	His or her contribution to mathematics	10 points
3.	Other interesting information	10 points

Total points (35 max) ______________.

The project is due ________________.

Late projects will not be accepted !!!!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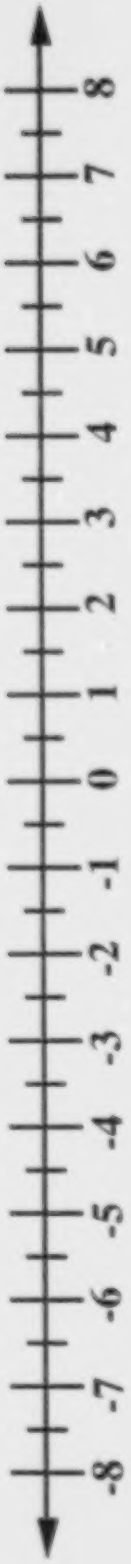
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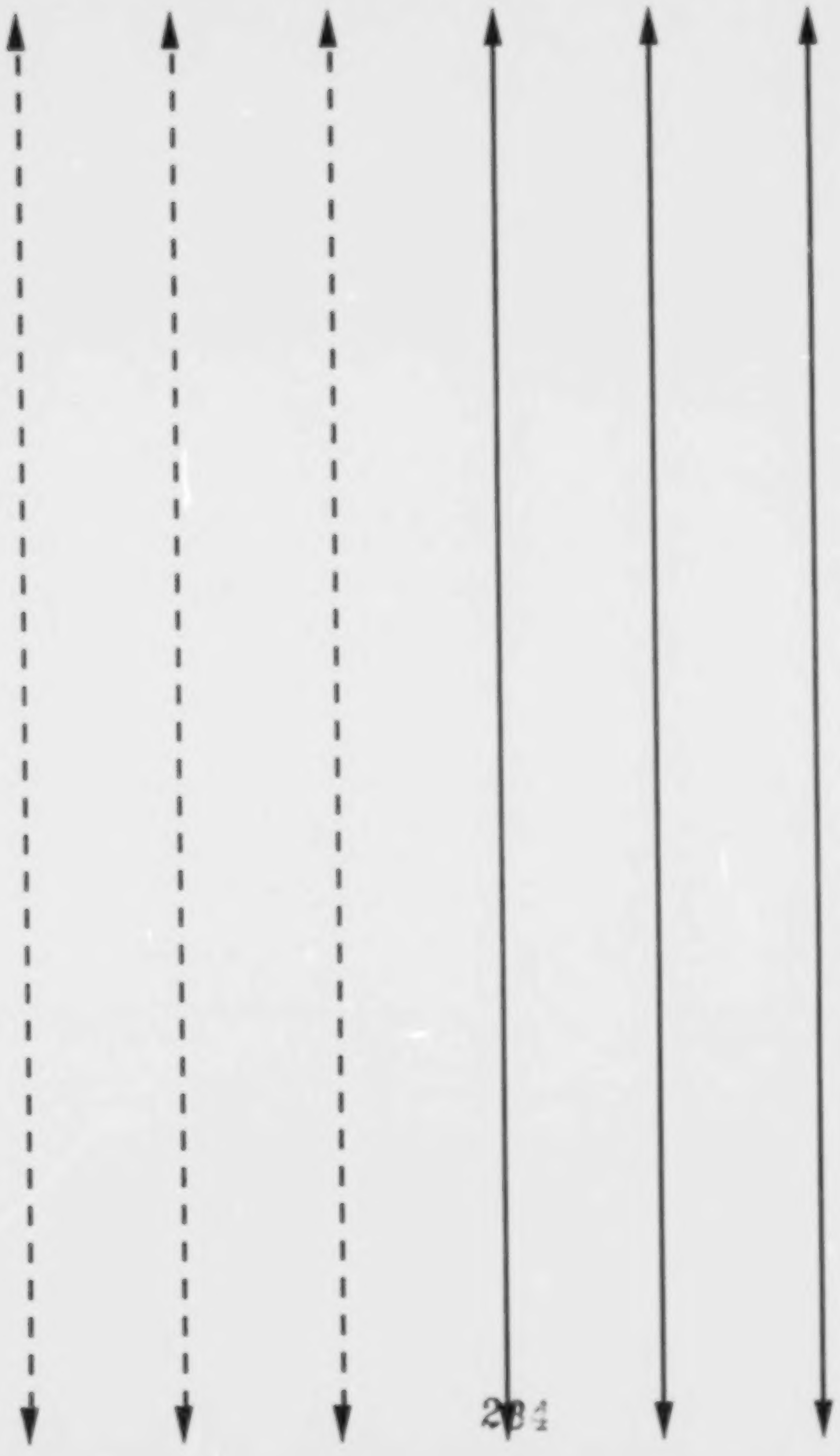
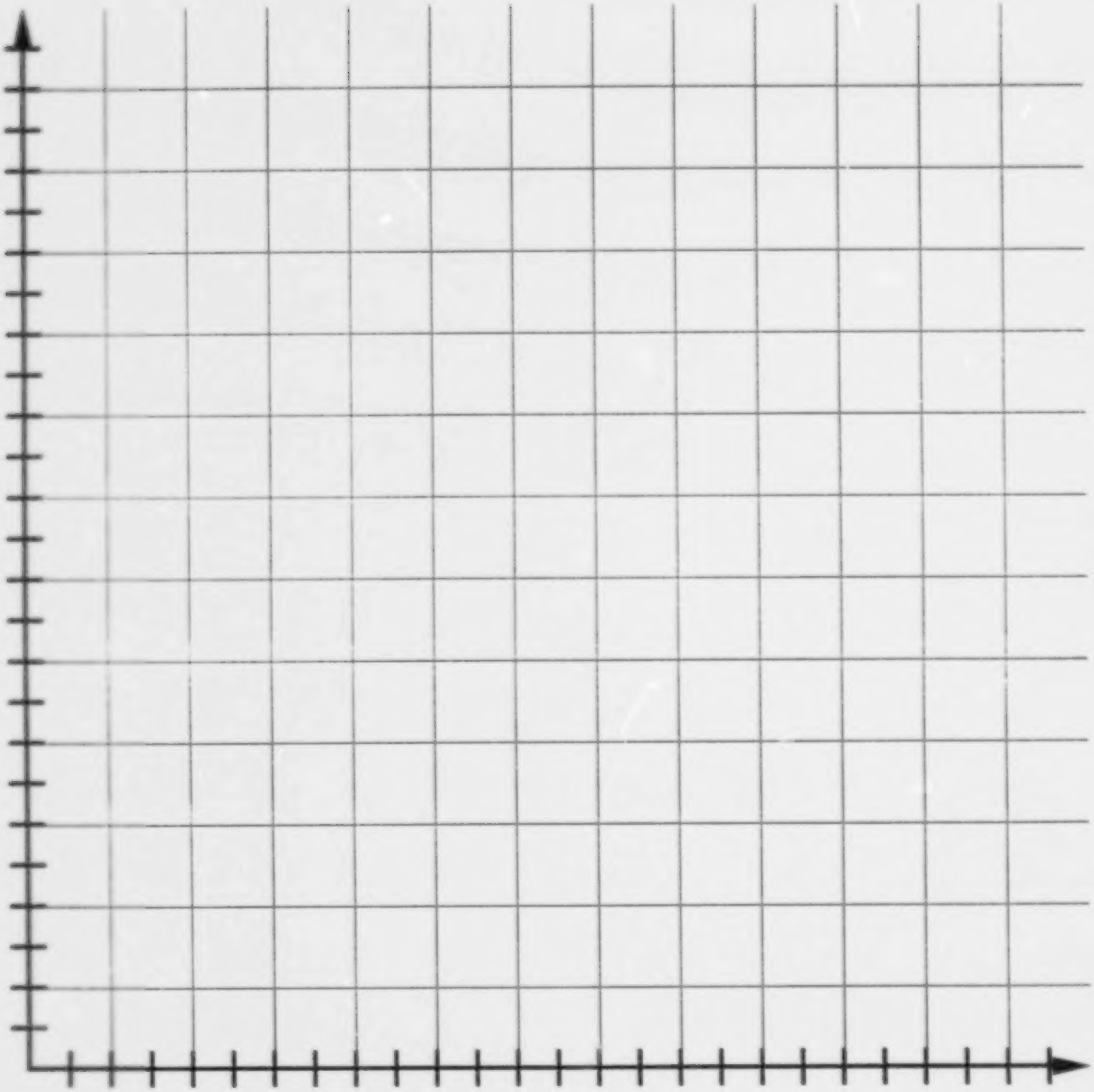
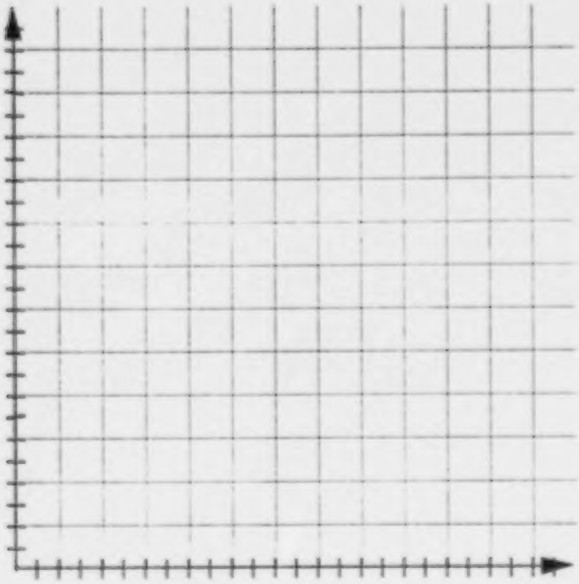
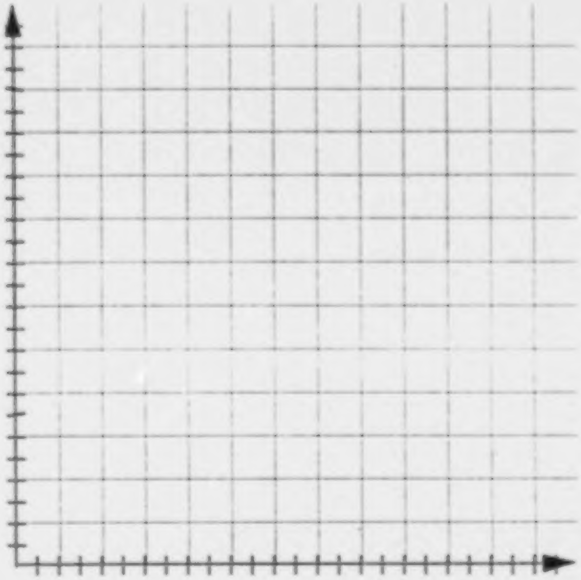
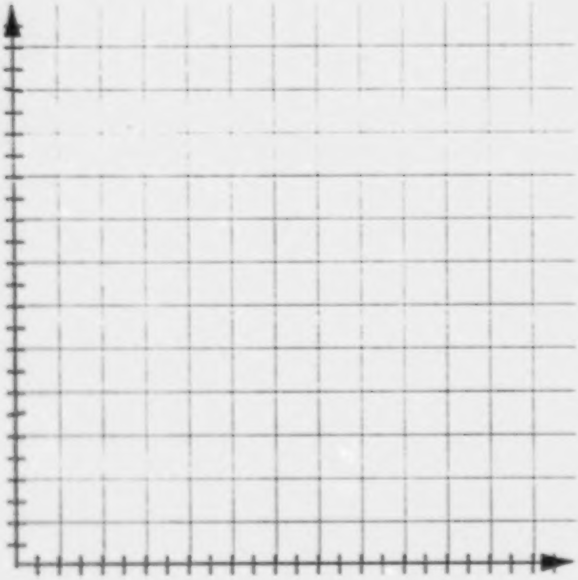
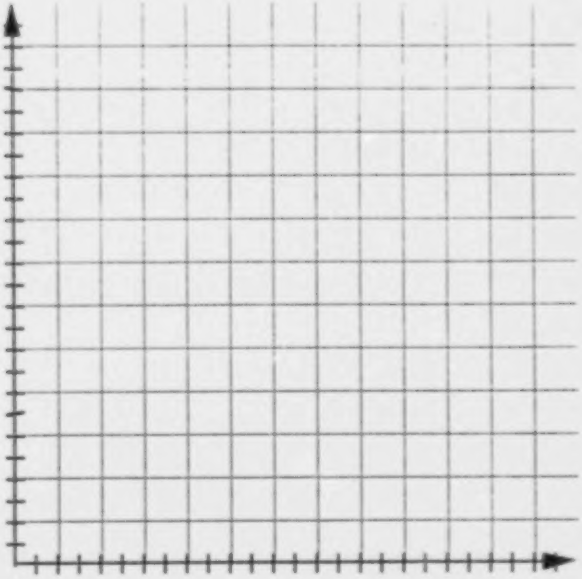


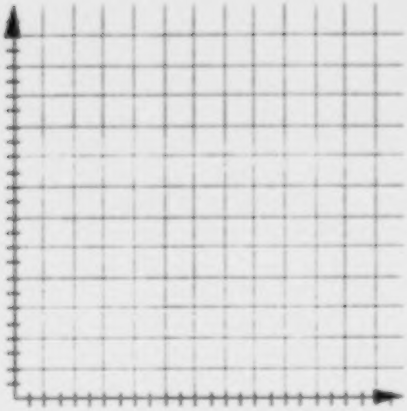


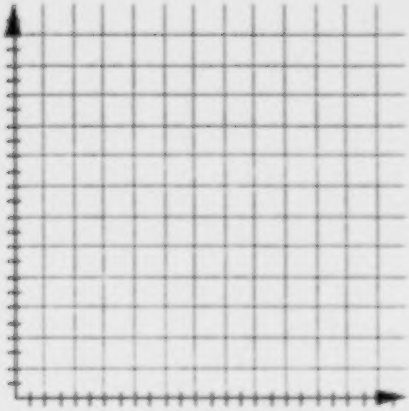


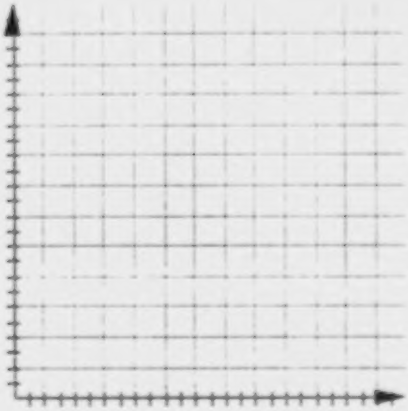


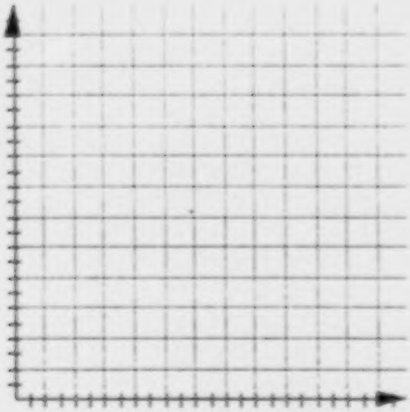


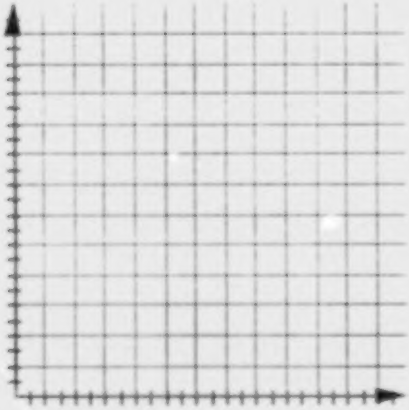


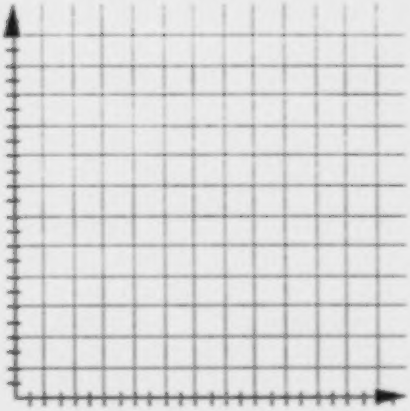


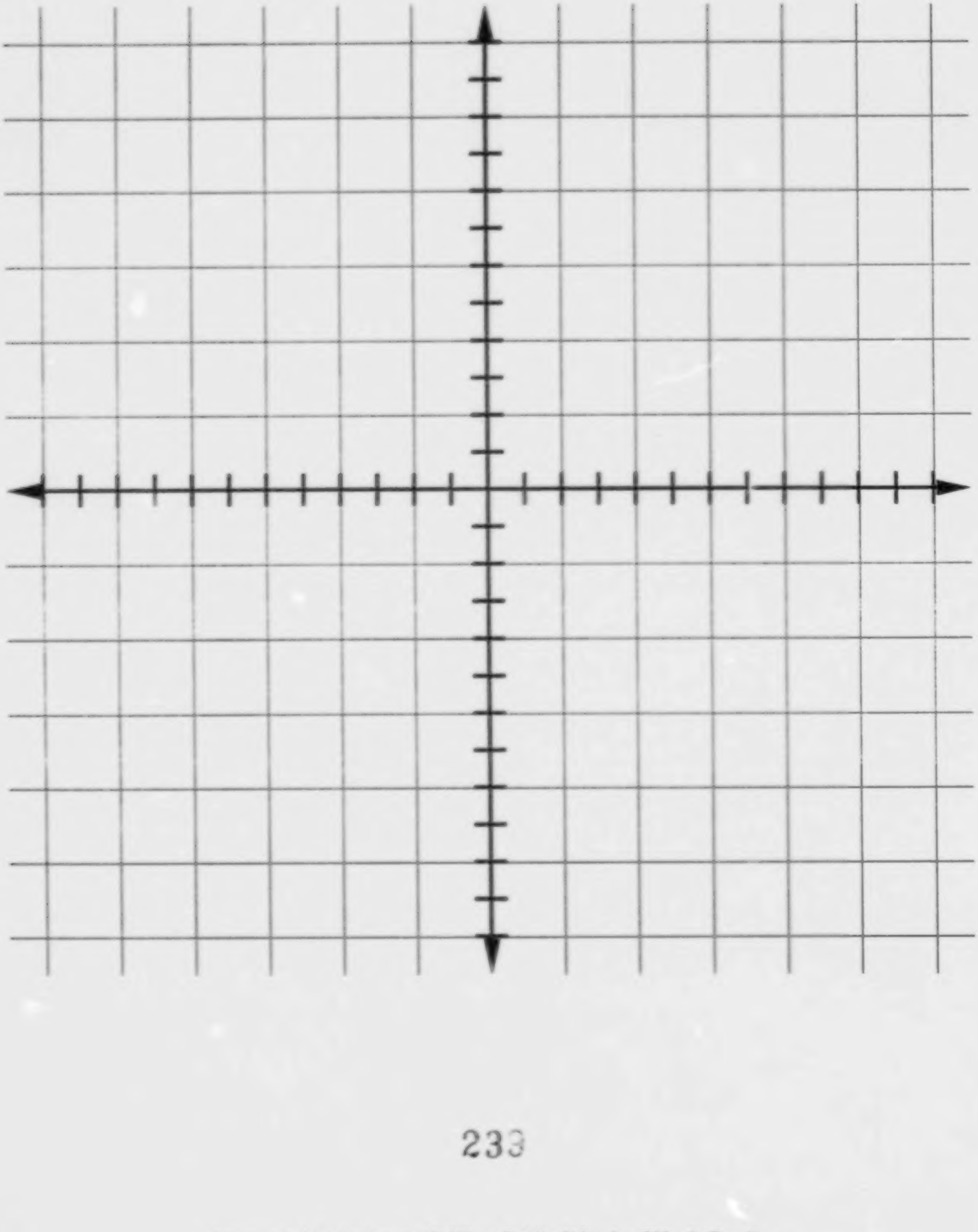


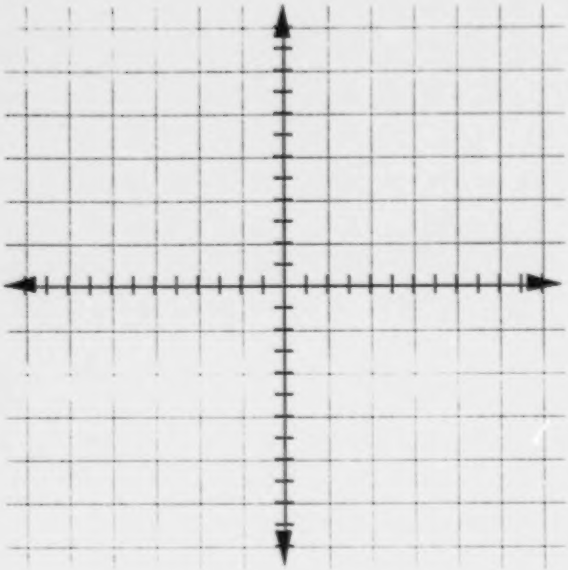


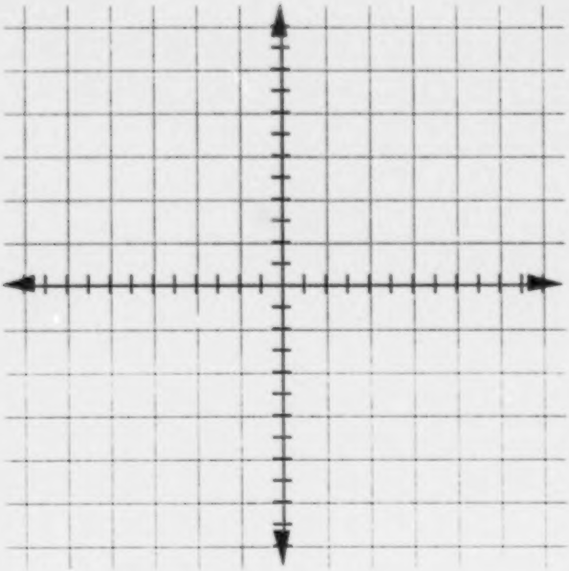


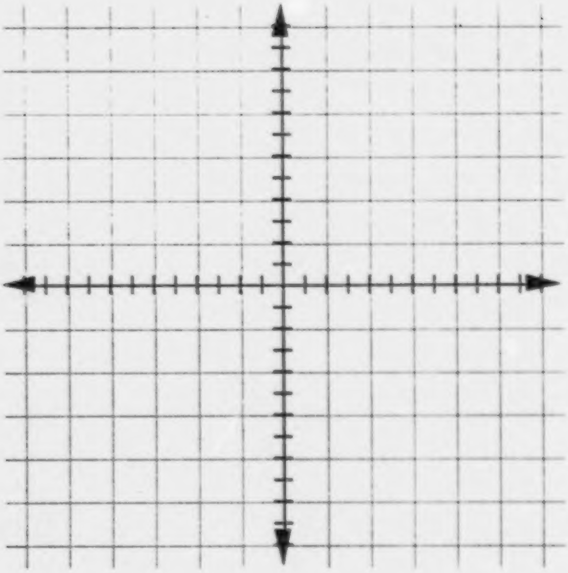


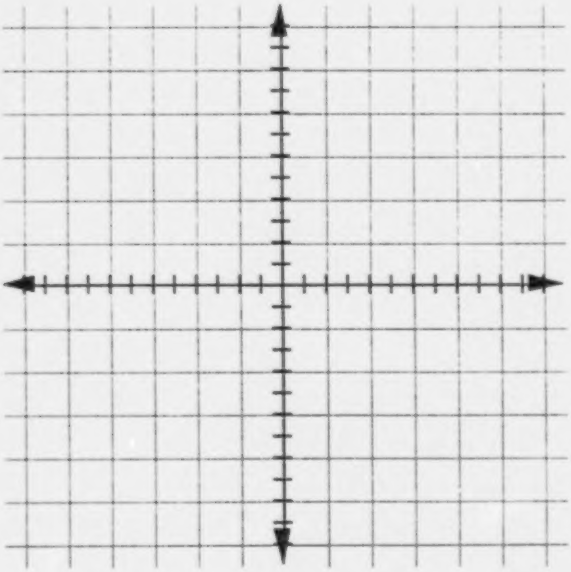


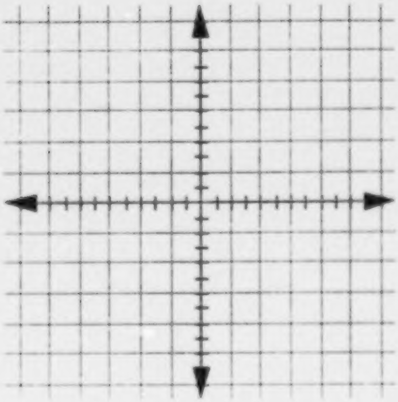


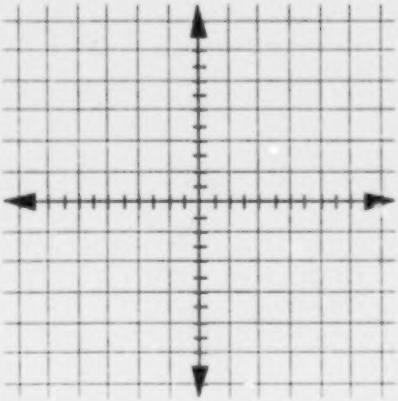


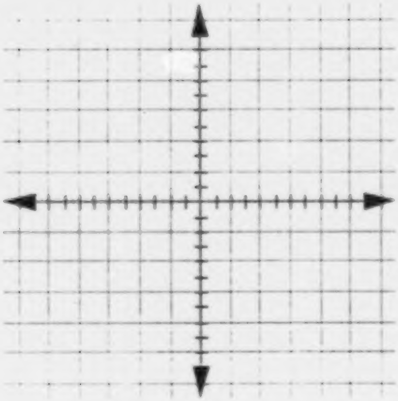


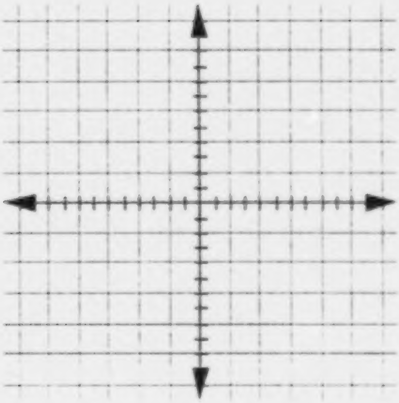


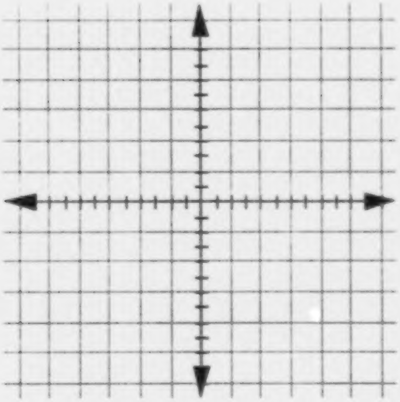


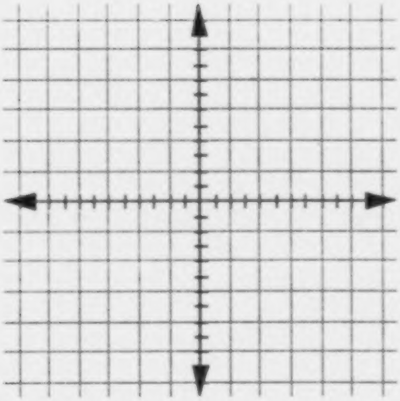


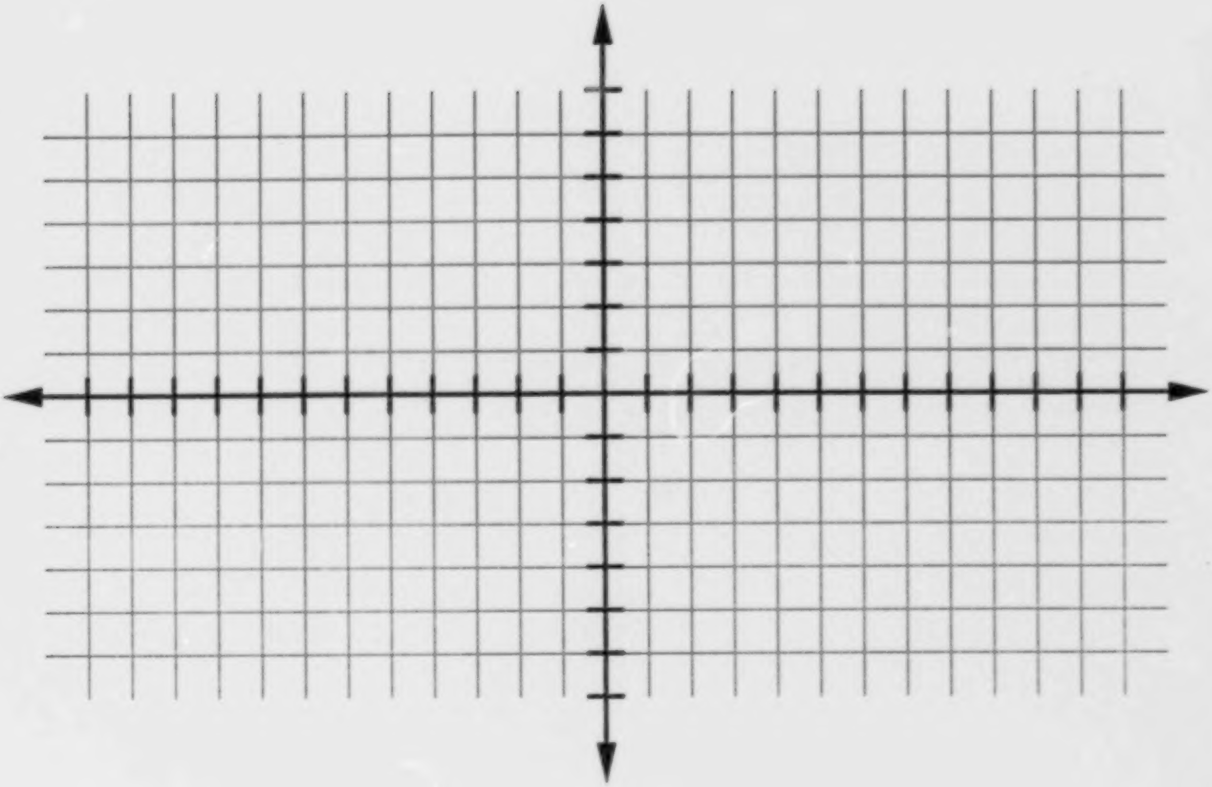


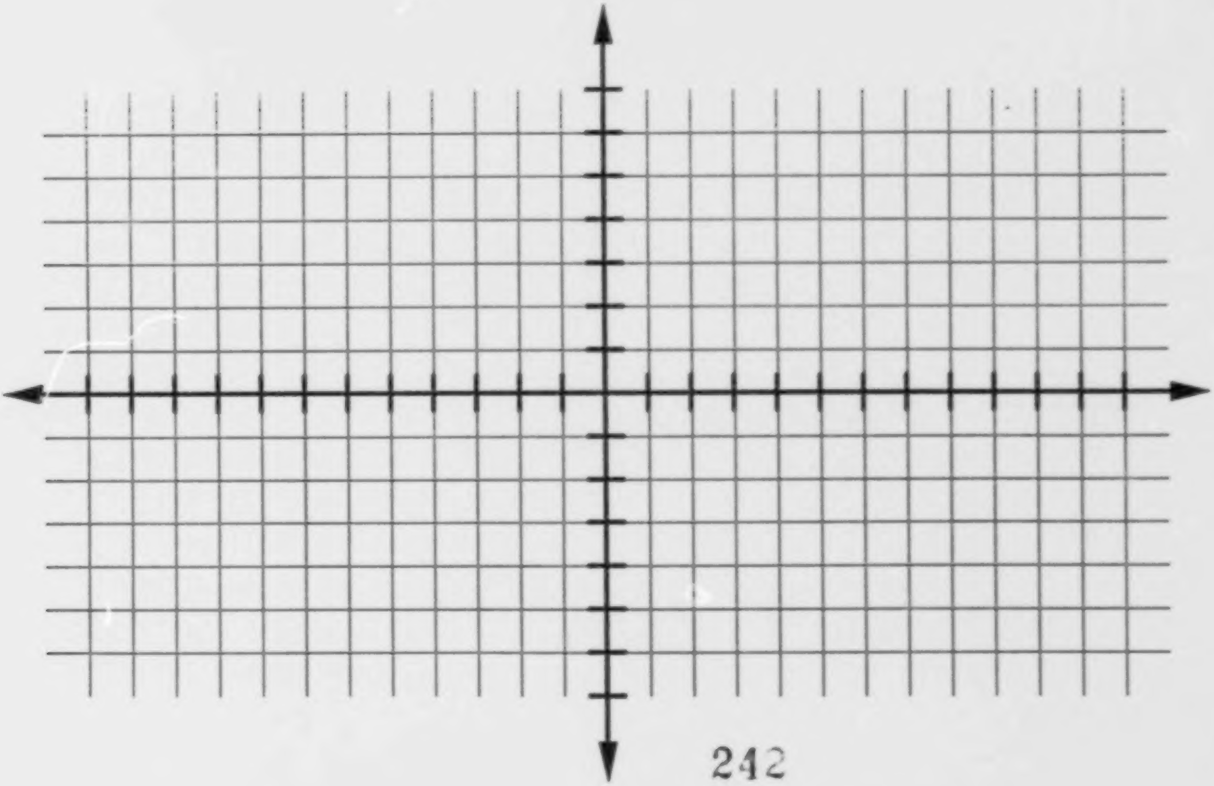












Number Tricks

Choose a number. ■

Add 5. ■ ○ ○ ○ ○ ○

Double the result. ■ ○ ○ ○ ○ ○
■ ○ ○ ○ ○ ○

Subtract 4. ■ ○ ○ ○
■ ○ ○ ○

Divide by 2. (Take half.) ■ ○ ○ ○

Subtract the number you started with ○ ○ ○

The result is ...

Basketball
Recording Sheet

Names ______________________________

____

Date __________

Period __________

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________
21. ________
22. ________
23. ________
24. ________
25. ________

Frequency Table

0 points ________

1 point ________

2 points ________

Total points scored ______________

Average points scored per game ______________

FT Shooting Percent ____________

Class Frequency Table

0 points ___________

1 point ___________

2 points ___________

Total points scored _______________

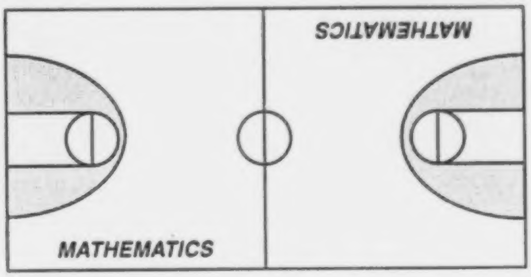
Average points scored _____________

FT Shooting Percent _______________

Basketball Extension 1: How good is your player?

Focus:	Graph and use linear equations.
Review:	Work with ratios, proportions, and percents.

Goal:
The students will simulate the free throw situation described in ***With the game on the line ...*** for different players on the Charlotte Hornets roster (or use players of your own choosing). The students will determine if there are any patterns exhibited in their results.



Materials:
Recording sheets (**B-32**), Charlotte roster (**B-33**), graph paper, calculators.

Description:
Students, working in pairs, are going to simulate the end-of-game situation 50 times and keep a record of their results. Students will use a random number generator (telephone book, calculator) to simulate shooting free throws. Students will compare their experimental results and discuss which players they would choose to be shooting free throws with the outcome of the game on the line. Students should be able to discuss the relationship between the shooting percentage and average points scored.

Procedure:
Assign teams of students a player from the Charlotte Hornets roster (**B-33**). Have them record the player's name, FT (free throws made), and FTA (free throws attempted) on the recording sheet (**B-32**). Compute FT/FTA (shooting percentage) and record.

Have the students follow the same procedure they used in **With the game on the line ...**

Have each team report their player's original shooting percentage (FT/FTA) and the average points scored. Make a table of those results.

Example:	**FT/FTA**	.58	.69	.78	.93
	Avg points scored	1.19	1.35	1.51	1.93

Have the students graph the ordered pairs (FT/FTA, AvgPts) and describe any patterns apparent.

Have the students find a line of best fit (with or without a graphing calculator). Determine the slope and y-intercept and explain within the context of the problem.

Expect an equation approximating **$y = 2x$**, where **x** is the free throw percentage and **y** is the average points scored. In the ideal situation where the slope is 2, we would say that the average points scored is twice the shooting percentage.

Basketball
Recording Sheet

Names ____________________

____

Date __________

Period __________

Player ______________ **FT =** ______ **FTA =** ______ **FT/FTA =** ________

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________
21. ________
22. ________
23. ________
24. ________
25. ________
26. ________
27. ________
28. ________
29. ________
30. ________
31. ________
32. ________
33. ________
34. ________
35. ________
36. ________
37. ________
38. ________
39. ________
40. ________
41. ________
42. ________
43. ________
44. ________
45. ________
46. ________
47. ________
48. ________
49. ________
50. ________

Frequency Table

0 points __________

1 point __________

2 points __________

Total points scored ______________

Average points scored per game ______________

FT Shooting Percent ______________

1995-96 Charlotte Hornets

Player	Min	G	FG	FGA	FT	FTA	3FG	3FGA	Reb	Ast	Stl	To	Blk
Rice	3142	79	610	1296	319	381	171	403	378	232	91	163	19
Johnson	3274	81	583	1225	427	564	67	183	683	355	55	182	43
Anderson	2344	69	349	834	260	338	92	256	203	575	111	146	14
Curry	2371	82	441	974	146	171	164	406	264	176	108	130	25
Burrell	693	20	92	206	42	56	37	98	98	47	27	43	13
Bogues	77	6	6	16	2	2	0	1	7	19	2	6	0
Adams	329	21	37	83	26	35	14	41	22	67	21	25	4
Parish	1086	74	120	241	50	71	0	0	303	29	21	50	54
Hancock	838	63	112	214	47	73	1	3	98	47	28	56	5
Geiger	2349	77	357	666	149	205	3	8	649	60	46	137	63
Goldwire	621	42	76	189	46	60	33	83	43	112	16	63	0
Zidek	888	71	105	248	71	93	0	0	183	16	9	38	7
Myers	1092	71	91	247	80	122	14	58	140	145	34	81	17
Addison	516	53	77	165	17	22	0	9	90	30	9	27	9
Glass	71	15	12	33	1	1	1	6	8	4	3	0	1
Hodge	115	15	9	24	0	0	0	0	23	4	1	2	8
Beck	33	5	2	8	1	2	0	0	7	5	1	4	0

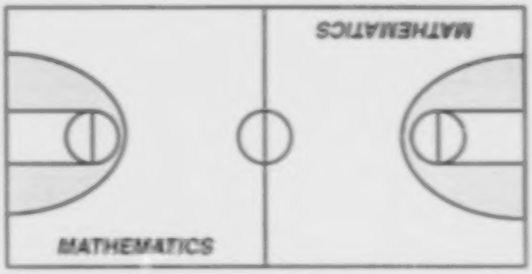
Basketball Extension 2: In theory, my player should ...

Focus: Use the language of algebra.
Review: Perform operations with real numbers.

Goal:
Determine theoretical probabilities for the various free throw shooting outcomes and relate average points scored to the expected value function.

Materials:
Calculators and results from ***With the game on the line ...*** and ***How good is your player?***

Description:
In ***With the game on the line ...***, the students calculated an average points scored. In probability, this result is called the **expected value**. Direct teaching and Socratic discussion should characterize this activity. Students should have an adequate background in probability from middle school mathematics.



Procedure:
According to the introductory problem, what was Curry's chance of hitting a free throw? (146/171) **What was his chance of missing a free throw?** (25/171)

Compare all of the following theoretical results with the experimental results from ***With the game on the line ...*** and ***How good is your player?***

Ask the students **what has to happen to score 0 points?** (MISS twice)
$P_0 = \text{MISS}\cdot\text{MISS} = (25/171)\cdot(25/171) = .0214$

Ask the students **what has to happen to score 1 point?** (HIT and MISS or MISS and HIT)
$P_1 = \text{HIT}\cdot\text{MISS} + \text{MISS}\cdot\text{HIT} = (146/171)\cdot(25/171) + (25/171)\cdot(146/171) = .2496$

Ask the students **what has to happen to score 2 points?** (HIT twice)
$P_2 = \text{HIT}\cdot\text{HIT} = (146/171)\cdot(146/171) = .7290$

Point out to students that (chance of a HIT) + (chance of MISS) = 1 and $P_0 + P_1 + P_2 = 1$. The sum of all outcomes (chances) has to be 1, otherwise all possibilities are not accounted.

Expected Value $= 0\cdot P_0 + 1\cdot P_1 + 2\cdot P_2 = 0\cdot(.0214) + 1\cdot(.2496) + 2\cdot(.7290) = 1.71$

Have the students compare 1.71 with their team results and the class results from ***With the game on the line ...*** Have the students calculate the expected value for their players from ***How good is your player?*** and comment.

Basketball Extension 3: It really looks like algebra.

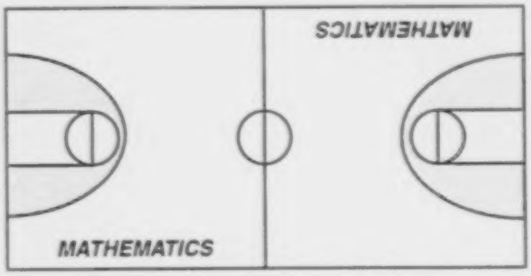
Focus: Perform operations with polynomials.
Review: Use the language of algebra.

Goal:
Use algebraic notation to generalize free throw shooting and explore the functions created.

Materials: Calculators and notes from ***In theory, my player should ...***

Description:
Direct teaching and Socratic discussion of variables, polynomials, and theoretical probability as it relates to the free throw shooting experiences should characterize this activity.

Procedure:
Using algebraic notation we can generalize free throw shooting and accommodate any player's statistics. Let **x** be the probability of any player hitting a free throw.



What is any player's chance of missing a free throw? (1-x)

For any player in the 2-shot free throw situation,

P_0 = MISS•MISS = $(1 - x)\cdot(1 - x) = 1 - 2x + x^2$
P_1 = HIT•MISS + MISS•HIT = $x(1 - x) + (1 - x)x = 2x(1 - x) = 2x - 2x^2$
P_2 = HIT•HIT = $x\cdot x = x^2$

Graph these probability functions. **What are their domain and range?**

Have the students compute the frequency at which their player scored 0, 1, and 2 points. Evaluate the probability functions for their player and compare with their experimental results.

Recall, if Expected Value

$$\begin{aligned}&= 0\cdot P_0 + 1\cdot P_1 + 2\cdot P_2\\ &= 0\cdot(1 - 2x + x^2) + 1\cdot(2x - 2x^2) + 2(x^2)\\ &= 2x\end{aligned}$$

How good a free throw shooter does a player need to be in order to score at least 1 point in a 2-shot situation?

Have students derive the probability functions for scoring 0, 1, 2 points, and expected value in one-and-one and 3-shot free throw situations.

For one-and-one: $P_0 = 1 - 2x + x^2$, $P_1 = x - x^2$, $P_2 = x^2$, EV $= x + x^2$

For 3-shot: $P_0 = 1 - 3x + 3x^2 - x^3$, $P_1 = 3x - 6x^2 + 6x^3$, $P_2 = 3x^2 - 3x^3$, $P_3 = x^3$, EV $= 3x$

Krypto

- Each student should have a copy of this recording sheet.
- With a deck of Krypto cards*, deal five cards to each student and one target card for the whole group.
- Record the five numbers that were dealt and the target number.
- Each student must use all five numbers and any operations to create an expression equal to the target number.

1. **Krypto Hand** **Target Number** ________

Expression ____

2. **Krypto Hand** **Target Number** ________

Expression ____

3. **Krypto Hand** **Target Number** ________

Expression ____

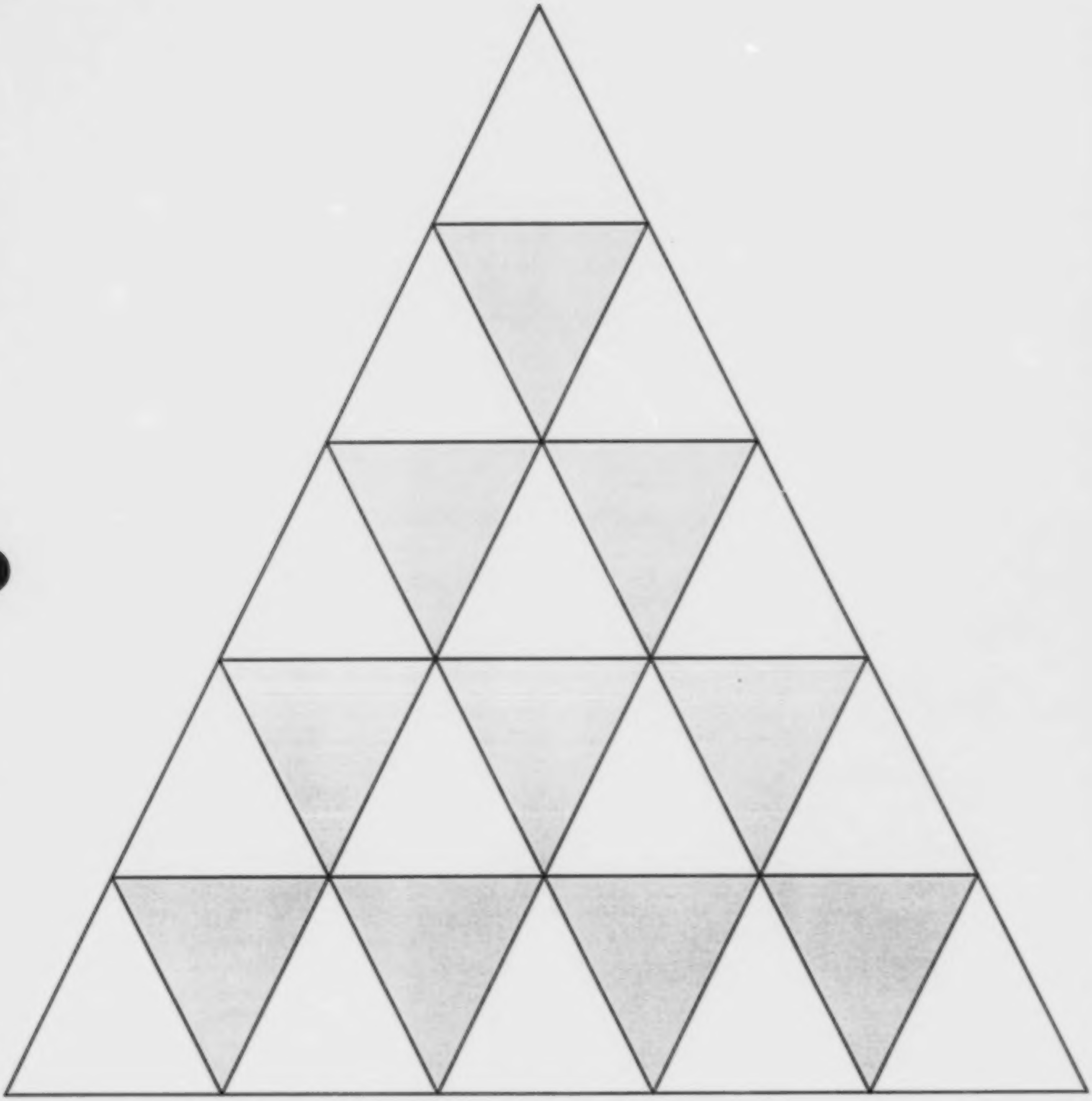
4. **Krypto Hand** **Target Number** ________

Expression ____

---

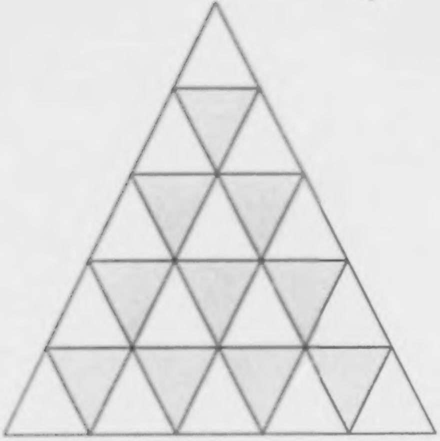
* Cards that are needed: three of each number 1 - 10; two of each number 11 - 17; and one of each number 18 - 25.

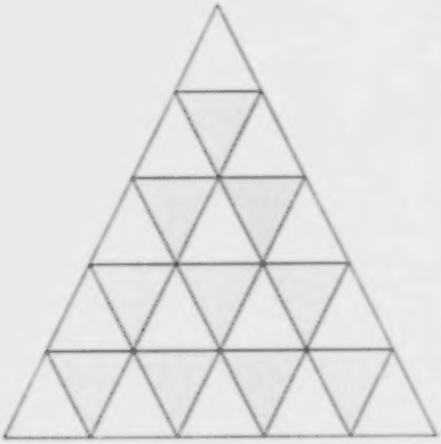
Race To The Top

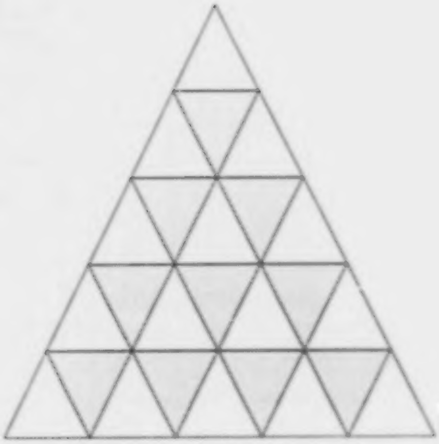


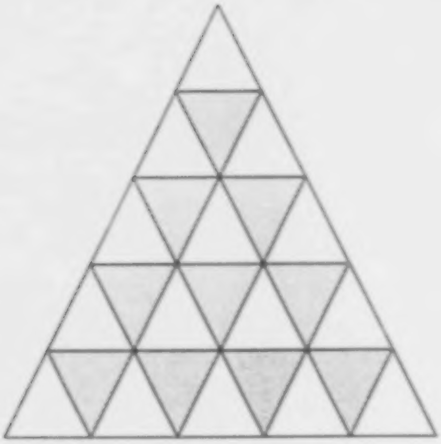
Race To The Top

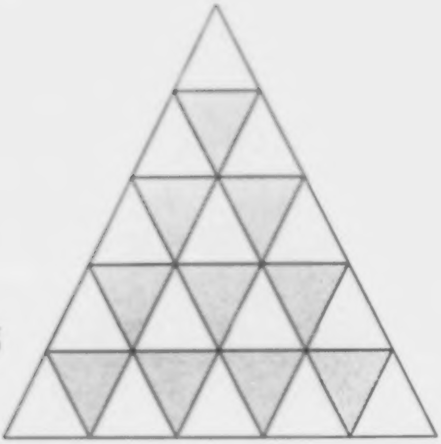
Name ______________________



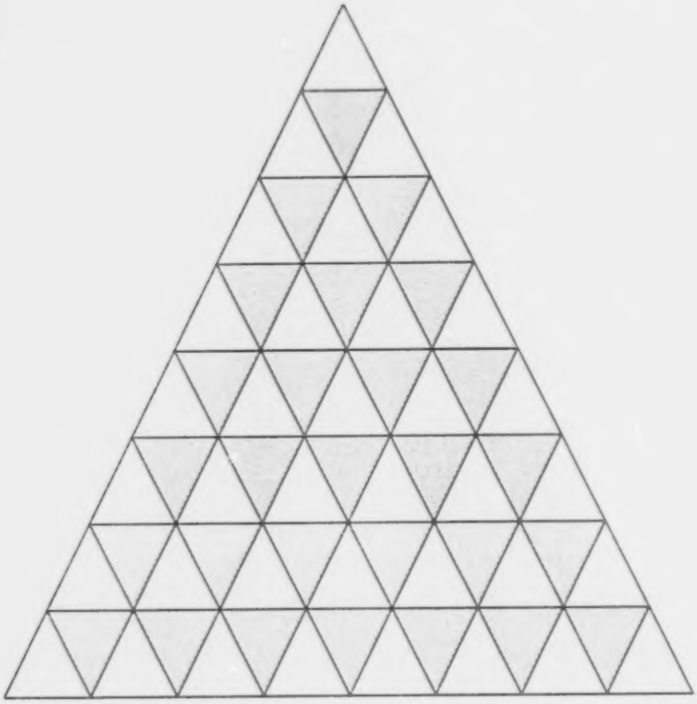


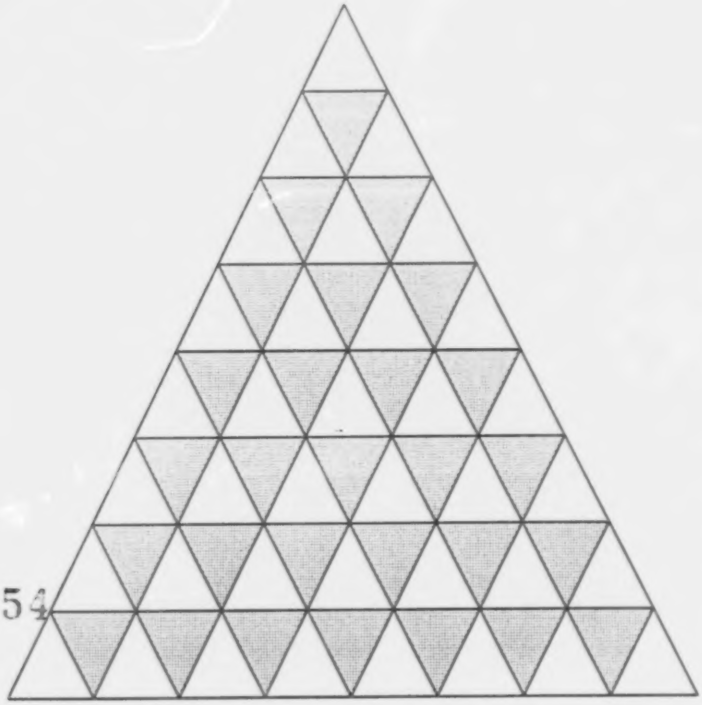






Race to the Top





Four In A Row

-24	-20	-18	-16	-15	free
-12	-10	-9	-8	-6	-5
free	-4	-3	-2	-1	1
2	3	4	5	6	8
9	10	12	15	16	18
20	24	free	25	30	36

To play ***Four in a Row***, you will need markers of two different shapes or colors and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly **ONE** clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally.

-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

Relays

1. Evaluate: $2 \bullet 3 + 5 \bullet 6 =$ __________
Put your answer in space #1.

2. Evaluate: $4 + 2(5 - 1) =$ __________
Add your answer to the quantity in space #1 and put the result in space #2.

3. Evaluate: $2 \bullet 3^2 =$ __________
Subtract your answer from the quantity in space #2 and place the result in space #3.

4. Evaluate: $3 + 4(1 + 2) =$ __________
Multiply your answer by 2 and add it to the quantity in space #3. Place your result in space #4.

5. Evaluate: $10 \bullet 2 \div 5 + 15 =$ __________
Add your answer to the quantity in space #4 and put the result in space #5.

6. Evaluate: $2(1 + 3)^2 =$ __________
Subtract your answer from the quantity in Space #5 and put the result in space #6.

Relay Form

1.	**4.**
2.	**5.**
3.	**6.**

Relays

1.

2.

3.

4.

5.

6.

Relay Form

1.

2.

3.

4.

5.

6.

257

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

Absolutely !!

Round	You	Opponent
1		
2		
3		
4		
5		
Totals		

Absolutely !!

Round	You	Opponent
1		
2		
3		
4		
5		
Totals		

Patterns With Exponents

I. Make a table and fill in values using exponents 1 through 10 for bases 2, 3, 4, 5, ... , 10. Use a spreadsheet or calculator.

Exponent	**Base 2**	**Base 3**	**Base 4**	**Base 5**	**etc.**
1	2	3	4	5	
2	4	9	16	25	
3	8	27	48	125	
4	16	81			
5	32	243			
6	64				
7	128				
8	256				
9	512				
10	1024				

II. Using the table, answer the following:

1. What are the possible digits in the units place for exponents of 2?

2. What are the possible digits in the units place for exponents of 3?

3. Find all possible units digits for:

Base

4 ______________________________

5 ______________________________

6 ______________________________

7 ______________________________

8 ______________________________

9 ______________________________

10 ______________________________

4. What would be the last digit of : (Use the above pattern)

a) 10^{25}

b) 6^{48}

c) 2^{15}

d) 3^{16}

e) 9^{25}

5. Which is larger? 6^{10} or 7^{10}

10^{5} or 8^{6}

4^{8} or 3^{9}

6. What base gives the following values:

$B^{7} = 823{,}543$; B=

$B^{6} = 531{,}441$; B=

$B^{5} = 371{,}293$; B=

7. What is the largest power of 2 that your calculator will display without using scientific notation?

8. What is the last digit of 24^{50} ?

From: Peters et al.. (1991). Patterns and Functions. Reston, VA: NCTM.

Number Crunching With Ease!

Scientific notation can make calculating with large and small numbers a breeze! In part A, complete the applications from science. In part B, you will see how you can use scientific notation on a calculator to solve problems.

A. 1. Our solar system is about $3 \cdot 10^4$ light years from the center of the Milky Way Galaxy. A light year is a distance approximately $5.9 \cdot 10^{12}$ miles. How many miles are we from the center of the Milky Way?

2. A poliomyelitis virus is about .025 microns in length. A micron is 10^{-3} millimeters. Rewrite the length of the virus in millimeters using scientific notation. If a microbiologist looks at the virus with an electron microscope that magnifies by 10^4 times, what is the length of the image of the virus observed?

3. A scientist is given a picture taken with an electron microscope that magnifies 10^4 times. She needs to determine if the microorganism is a yeast or bacteria. The width of the organism in the photo is 48 millimeters. She knows that bacteria have a width approximately .5 microns while yeast are approximately 5 microns (a micron is 10^{-3} millimeters). Is the organism a yeast or bacteria? Explain how you can determine this?

4. Could a microbiologist use a light microscope that magnifies an image 1000 times to observe a virus of length .04 microns? What would be the length in millimeters of the image of the virus?

B. Use a calculator to complete the following:

1. Most calculators cannot calculate a number as large as 5^{150}. How could you use scientific notation and a calculator to calculate this number?

2. You are offered a job in which you are in training during the first year. Your boss offers to pay you $.01 the first week, $.02 the second week, $.04 the third week, $.08 the fourth week, etc. What would be the total amount that you would earn the first year?

   a. Complete the table

Week	Total Earned
1	
2	
3	
4	
5	

   b. Now, calculate and compare
   2^1, 2^2, 2^3, 2^4, 2^5

   c. Write a formula to calculate the total earnings at the end of 52 weeks. Calculate this answer on your calculator. Write the number in scientific notation and then in expanded form.

Bull's Eye Score Sheet

Goal Number: ________

Round	Result of Roll	Operation Chosen (add or subtract)	Cumulative Total
			0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			

In-Line Skating

(Calories consumed per minute by weight and speed)

Speed (mph)	8	9	10	11	12	13	14	15
Weight (lbs.)								
120	4.2	5.8	7.4	8.9	10.5	12.1	13.6	15.2
140	5.1	6.7	8.3	9.9	11.4	13.0	14.6	16.1
160	6.1	7.7	9.2	10.8	12.4	13.9	15.5	17.1
180	7.0	8.6	10.2	11.7	13.3	14.9	16.4	18.0
200	7.9	9.5	11.2	12.6	14.2	15.8	17.3	18.9

Cable Subscribers

(millions)

1965	1.5
1970	5.1
1975	9.8
1976	11.0
1977	12.2
1978	13.4
1979	15.0
1980	17.5
1981	21.5
1982	25.4
1983	29.45
1984	32.85
1985	35.43
1986	38.74
1987	41.2
1988	44.2
1989	47.39

Land Speed Records

(MPH)

1906	127.659
1910	131.724
1911	141.732
1919	149.875
1920	155.046
1926	170.624
1927	203.790
1928	207.552
1929	231.446
1931	246.086
1932	253.96
1933	272.109
1935	301.13
1937	311.42
1938	357.5
1939	368.9
1947	394.2
1963	407.45
1964	536.71
1965	600.601
1970	622.407
1983	633.6

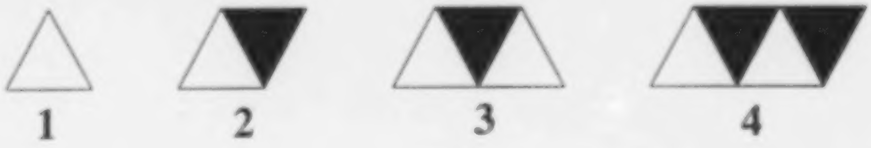
Personal Fitness

(calories used per minute by body weight)

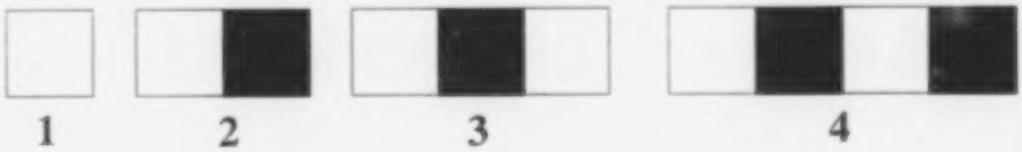
Weight (lbs)	100	120	150	170	200	220
Running (8 min miles)	9.4	11.3	14.1	16.0	18.8	20.7
Skiing (cross country)	7.2	8.7	10.8	12.3	14.5	15.9
Jogging (11 min miles)	6.1	7.3	9.1	10.4	12.2	13.4
Bicycling (10 mph)	5.4	6.5	8.1	9.2	10.8	11.9
Walking (4 mph)	3.9	4.6	5.8	6.6	7.8	8.5
Skating (moderate)	3.6	4.3	5.4	6.1	7.2	7.9
Swimming	5.8	6.9	8.7	9.8	11.6	12.7

Patterns in Perimeter

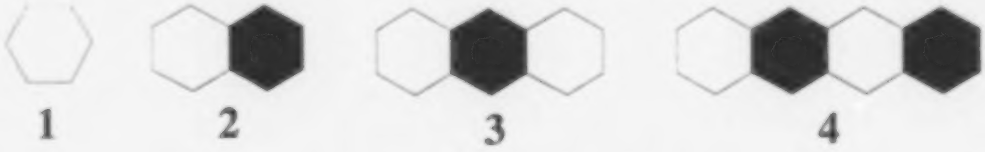
If each side of the first figure in each set has a length of one unit, what is the perimeter of the other figures in each set? Fill in the table accompanying each set of figures.



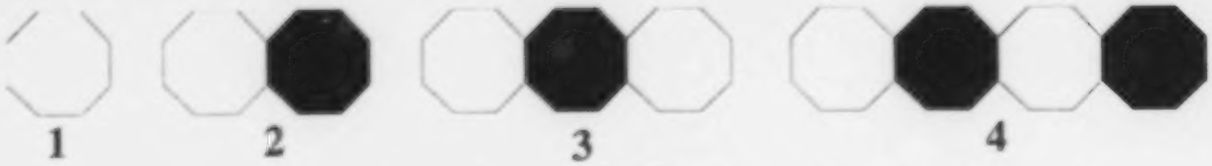
I. Triangles	1	2	3	4	5		10			n
Perimeter										



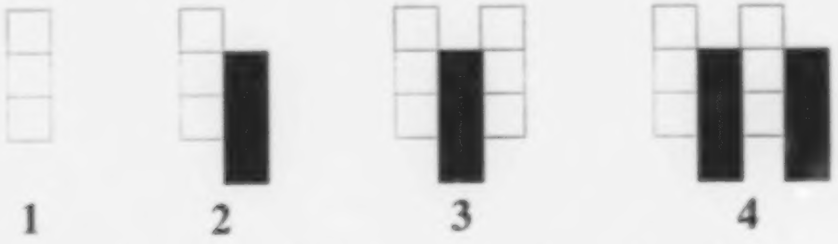
II. Squares	1	2	3	4	5		10			n
Perimeter										



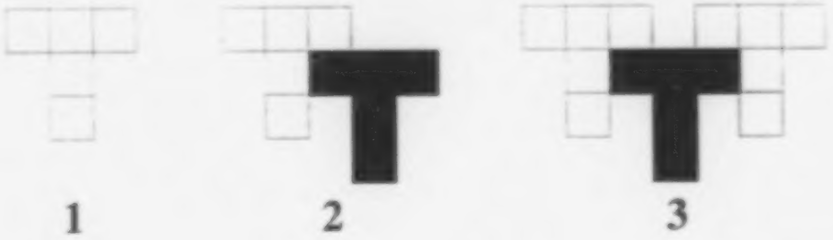
III. Hexagons	1	2	3	4	5		10			n
Perimeter										



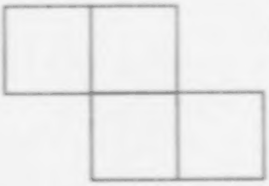
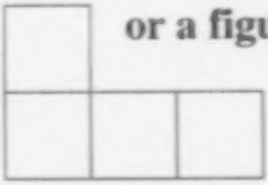
IV. Octagons	1	2	3	4	5		10			n
Perimeter	8	14							440	



V. 3-Squares	1	2	3	4	5		10			n
Perimeter	8								256	

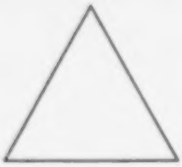


VI. T-Squares	1	2	3	4	5		10			n
Perimeter	12								268	

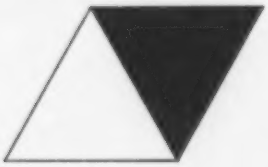
VII. Use  **or**  **or a figure of your own**

to make a sequence of shapes to be shared and analyzed.

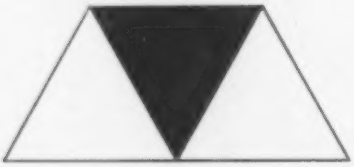
If the length of each side of the triangle is one unit, what are the perimeters of each of these figures?



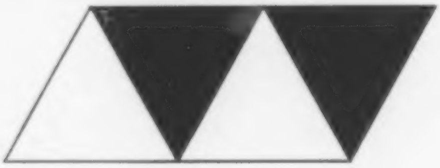
perimeter =



perimeter =



perimeter =

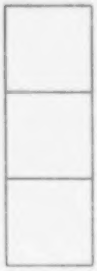


perimeter =

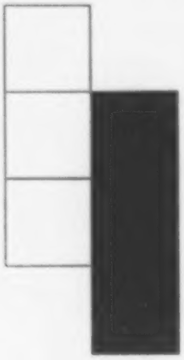
Triangles	1	2	3	4	5	6
Perimeter						

How does the perimeter change as the number of triangles changes?

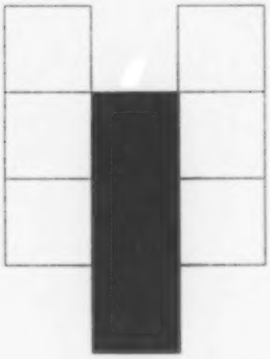
If the length of each side of a square is one unit, what are the perimeters of each of these figures?



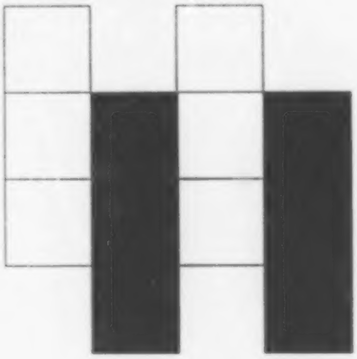
perimeter =



perimeter =



perimeter =



perimeter =

3-Squares	1	2	3	4	5	6
Perimeter						

How does the perimeter change as the number of 3-squares changes?

The Picture Tells the (Linear) Story

Use your graphing calculator to investigate each family of equations.
Sketch each equation's graph on the axes provided.
Answer the questions for each family of equations.

I.

y = x

y = x + 6

y = x - 4

- How are the lines the same?
- What is different about the lines?
- Where does each line cross the y-axis?
- What happens to the graph when a constant is added to y = x?
- Write an equation for a line similar to those above but crosses the y-axis at 5.
- Write an equation for a line similar to those above but crosses the y-axis at -2.

II.

y = x

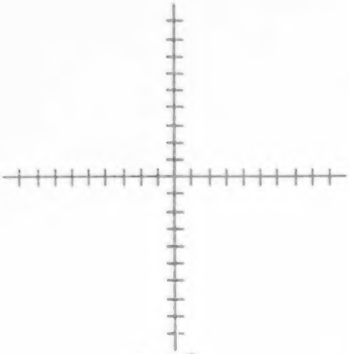
y = -x

- How are the lines alike?
- How are the lines different?

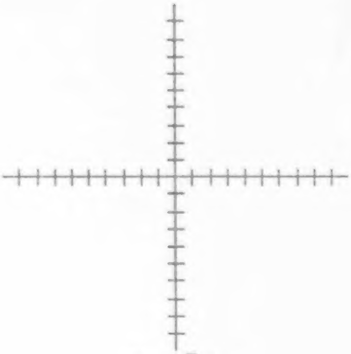
The Picture Tells the (Linear) Story

III.

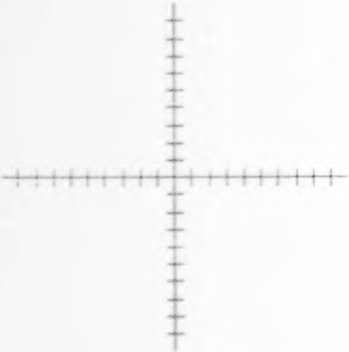
y = x



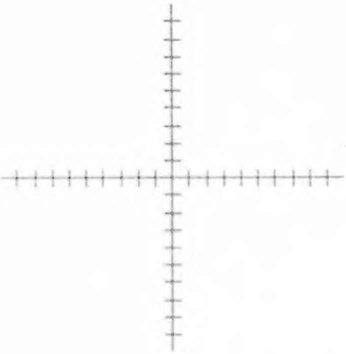
y = 2x



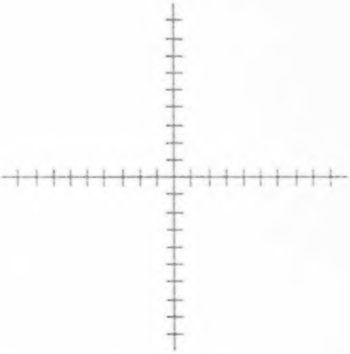
y = 5x



y = (1/2)x



y = (1/3)x



y = (1/4)x

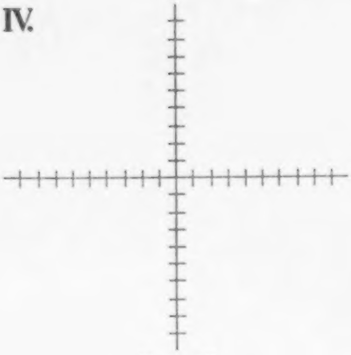
- Describe the differences in the graphs.

- Which line appears the steepest?

- What makes the difference?

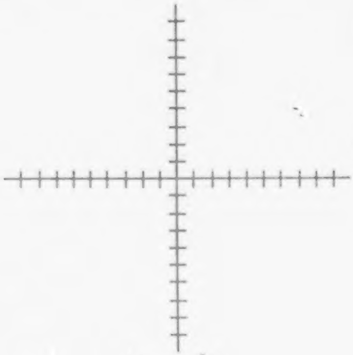
The Picture Tells the (Linear) Story

IV.



$y = -x$

$y = -2x$



$y = -4x$

- How are the lines different?

- Which line appears the steepest?

- What makes the difference?

V.

- Where does each of the following cross the y-axis?
  $y = 2x + 7$ ________________
  $y = -x + 11$ ________________
  $y = (1/2)x - 8$ ________________

- Which line is steepest and why.
  $y = x + 8$ ________________
  $y = 3x - 4$ ________________
  $y = (1/2)x + 3$ ________________

- Which line is steepest and why.
  $y = -x + 8$ ________________
  $y = -2x + 5$ ________________
  $y = -(1/3)x$ ________________

- If a linear equation can be written in the form $y = mx + b$, where **m** and **b** represent any real values, explain the effect of **m** on the graph of the equation.

- Explain the effect of **b** on the graph.

Guess My Slope and Y-intercept for the TI-81/82

Here is a program that will draw a line and allow the student to guess the slope and y-intercept. The calculator will then draw the line for the guessed slope and y-intercept. If the graphs match, the student receives 5 points. If the line is incorrect, the opponent can then try to guess the correct slope and y-intercept.

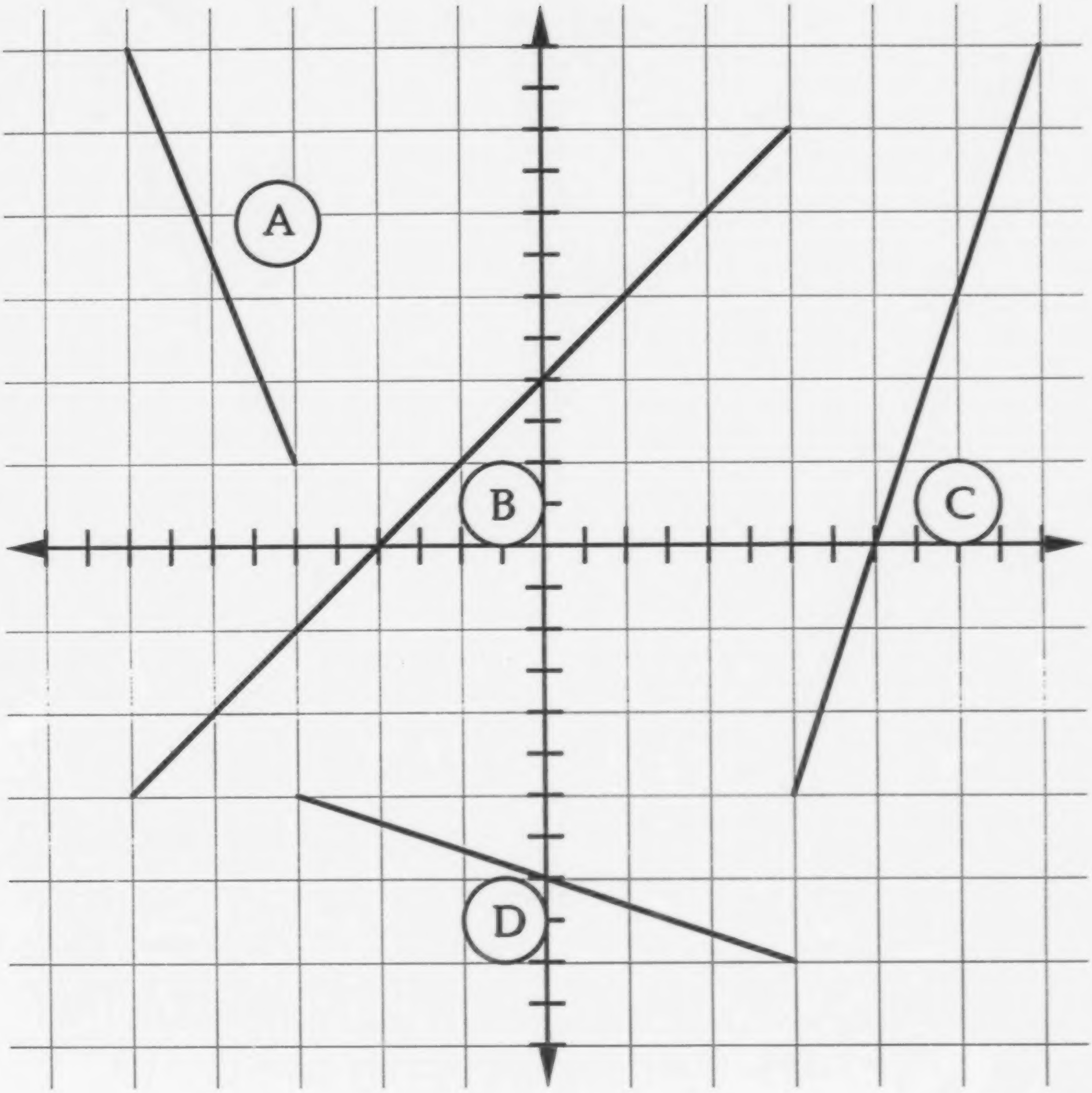
```
0 STO M
0 STO B
Grid On
"CX + D" STO Y1
IPart (8Rand - 3) STO P
IPart (3Rand + 1) STO D
P/D STO C
IPart (8Rand - 3) STO D
-6 STO Xmin
6 STO Xmax
-4 STO Ymin
4 STO Ymax
1 STO Xscl
1 STO Yscl
DispGraph
Pause
Lbl 0
Disp "SLOPE?"
Input M
Disp "Y INTERCEPT?"
Input B
DrawF MX + B
Pause
Disp "TRY AGAIN?"
Input A
If A=1
Goto 0
DispHome
```

1. Begin the program. The calculator will display a graph then pause. (The first fourteen lines randomly generate a slope and y-intercept for an equation with range, $-6 \le x \le 6$ and $-4 \le y \le 4$). Press **ENTER** when you are ready to enter the slope and y-intercept. When the calculator asks for slope, type in the slope. When asked for the y-intercept, type in the y-intercept. The calculator will now graph the line you have described.

2. Check to see if the new equation is the same line or not.

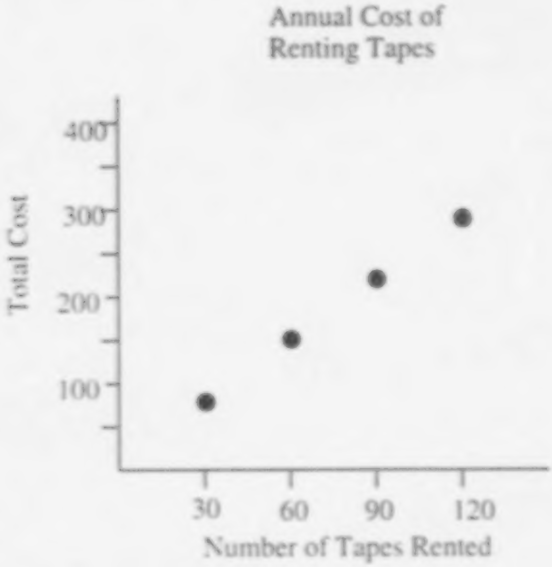
3. Type 1 for yes and **ENTER** if your line is correct. Type any number for no.

4. To play again, **PRGM** and select the program.


A
B
C
D

Making Sense of Slope

1. Viper Video charges a membership fee per year in addition to the rental cost per tape. The graph represents the annual cost as it varies with the number of tapes rented.



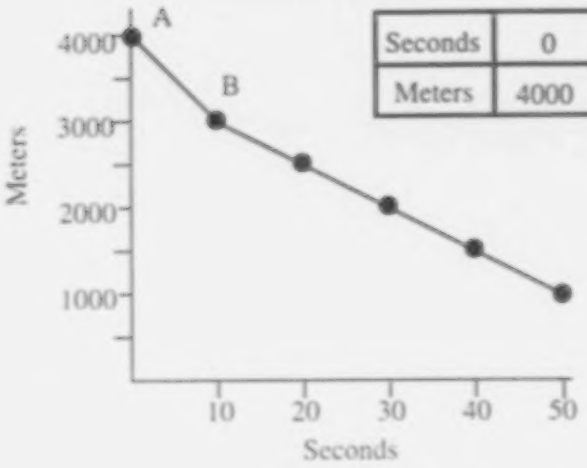
Tapes	0	30	60	90	120
Cost	7	82	157	232	307

a) Calculate the slope. Explain what the slope represents in terms of number of tapes rented and cost.

b) What does the y-intercept represent in this application?

2. A skydiver, during a jump, falls at a constant rate after the first ten seconds. The graph below shows the altitude of the skydiver after x seconds.

Altitude over Time



Seconds	0	10	20	30	40	50
Meters	4000	3000	2500	2000	1500	1000

a) Calculate the slope of the line after the first 10 seconds after point B). Explain the meaning of the slope in terms of meters and seconds.

b) What is the meaning of the y-intercept in this application?

c) What is the meaning of the x-intercept?

d) Why might there be a steeper slope from point A to point B?

3. The data below represent the value of a delivery truck as it depreciates over a period of 5 years.

Years	0	1	2	3	4	5
Truck Value	15000	12400	9800	7200	4600	2000

a) Graph the data.

b) Calculate the slope. What is the meaning of the slope in terms of years and current value?

c) What is the meaning of the y-intercept?

d) Write an equation to describe the data.

4. The data below represent the distance traveled by a pontoon boat every 10 minutes.

Minutes	10	20	30	40	50
Distance	4	8	12	16	20

a) Graph the data.

b) Calculate the slope. Describe the slope in terms of distance and minutes. Now, describe the slope in terms of miles and hours.

c) Write the equation describing the data in minutes. Write an equation in hours.

d) What would be the distance traveled after 1.5 hours?

5. Write an application for the equation: $y = 3.5x + 4$

a) Complete a table and graph.

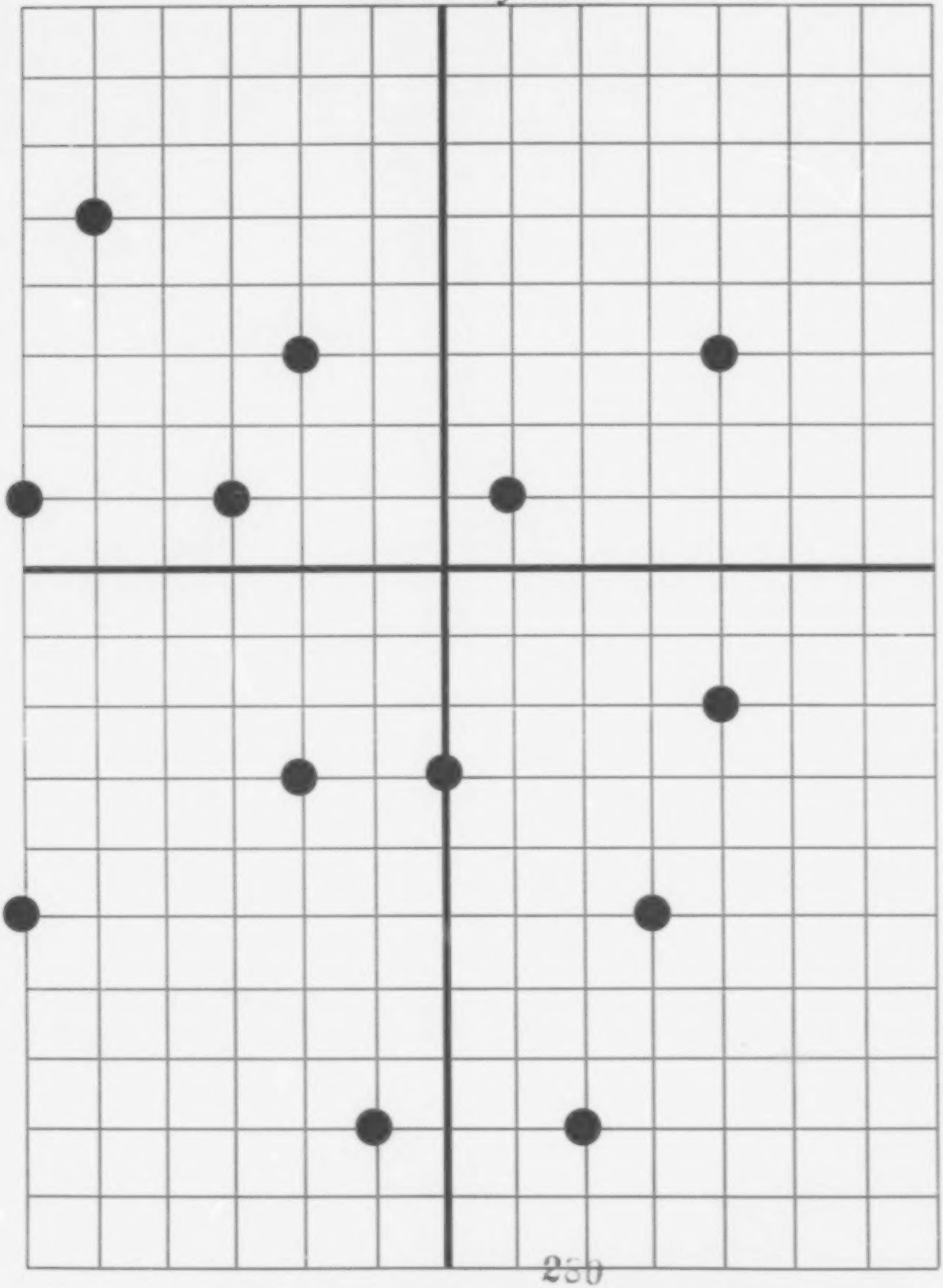
b) Interpret the meaning of the slope.

c) Interpret the y-intercept.

NBA 1995-96
Central Division, Eastern Conference

	Dec 2	Jan 1	Feb 7	Mar 3	April 3	Final
Atlanta	9-7	13-14	25-21	32-24	41-31	46-36
Charlotte	7-10	14-16	21-24	28-28	37-35	41-41
Chicago	13-2	24-3	41-5	51-6	63-8	72-10
Cleveland	6-9	14-12	26-20	33-23	41-31	47-35
Detroit	6-9	15-14	23-22	29-26	39-32	46-36
Indiana	6-7	15-12	31-16	37-20	44-29	52-30
Milwaukee	4-10	10-16	18-27	20-35	22-50	25-57
Toronto	6-10	9-21	13-34	14-41	19-53	21-61

Where's My Line?



North Carolina Population

1790	393,751
1800	478,103
1810	555,500
1820	638,829
1830	737,987
1840	753,419
1850	869,039
1860	992,622
1870	1,071,361
1880	1,399,750
1890	1,617,949
1900	1,893,810
1910	2,206,287
1920	2,559,123
1930	3,170,276
1940	3,571,623
1950	4,061,929
1960	4,556,155
1970	5,084,411
1980	5,880,095
1990	6,628,637

US Postal Rates

1958	4¢
1963	5¢
1968	6¢
1971	8¢
1974	10¢
1975	13¢
1978	15¢
1981	18¢
1985	22¢
1988	25¢
1991	29¢
1995	32¢

Olympic Swimming: 400 Meter Free-Style

(all times in seconds)

Year	**Men**	**Women**
1924	304.2	362.2
1928	301.6	342.8
1932	288.4	328.5
1936	284.5	326.4
1948	281.0	317.8
1952	270.7	312.1
1956	267.3	294.6
1960	258.3	290.6
1964	252.2	283.3
1968	249.0	271.8
1972	240.27	259.04
1976	231.93	249.89
1980	231.31	248.76
1984	231.23	247.10
1988	226.95	243.85
1992	225.00	247.18

1. Describe each set of data, identifying any patterns.

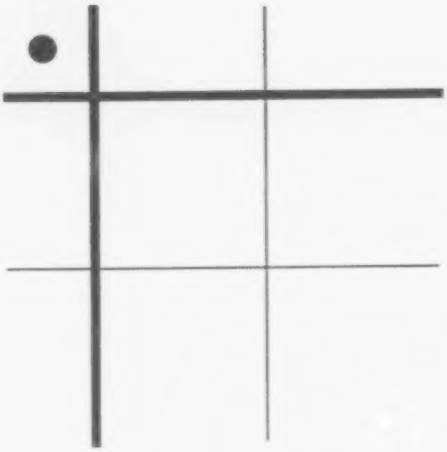
2. How have the times changed over the years? Why?

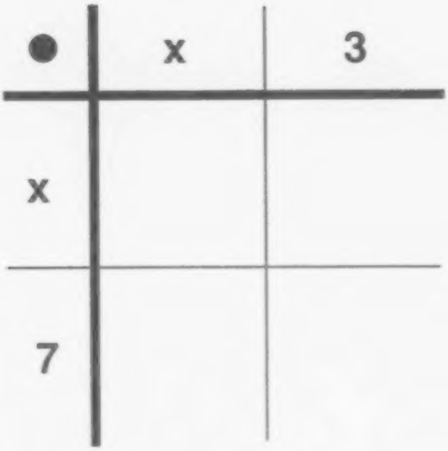
3. Derive functions of best fit for the men's and women's data.

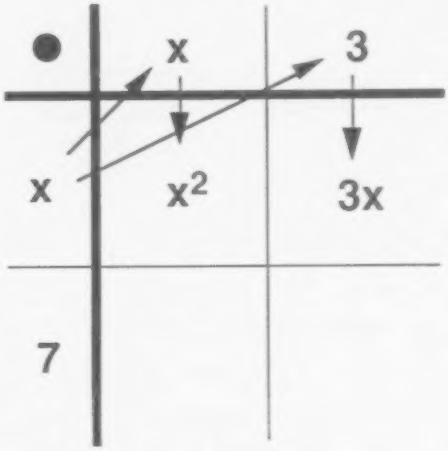
4. Using the functions, predict the results for the 1940, 1944, 1996, and 2020 Olympics. For which years are the predictions most reliable? Explain.

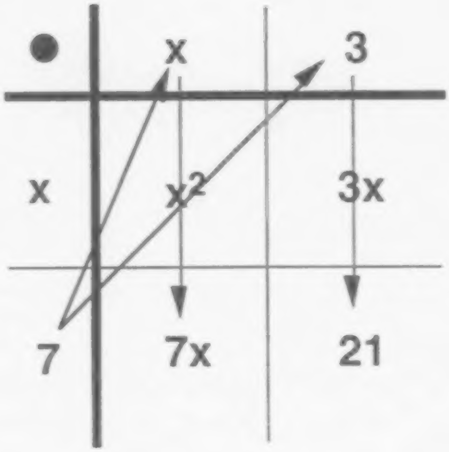
5. How did the 1996 results compare with the results predicted in #4?

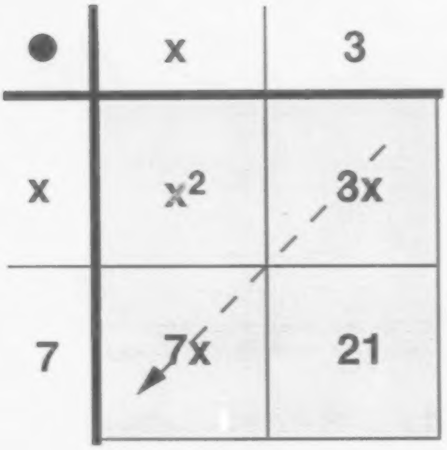
6. According to the functions, is it possible that a woman will swim faster than a man in this event? When? How likely is it?








x
3
x
x²
3x
7
7x
21

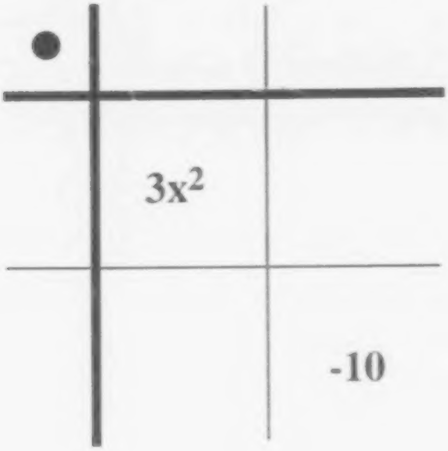

x
3
x
x²
3x
7
7x
21

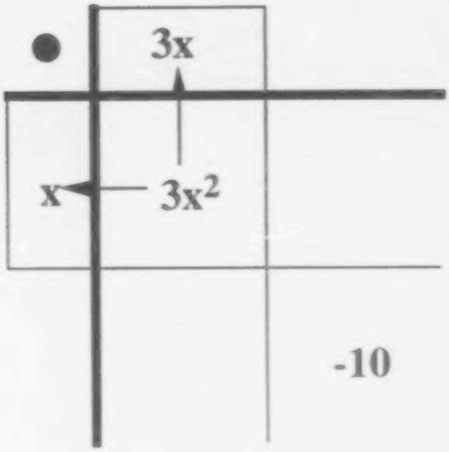
$$(x + 3)(x + 7) =$$

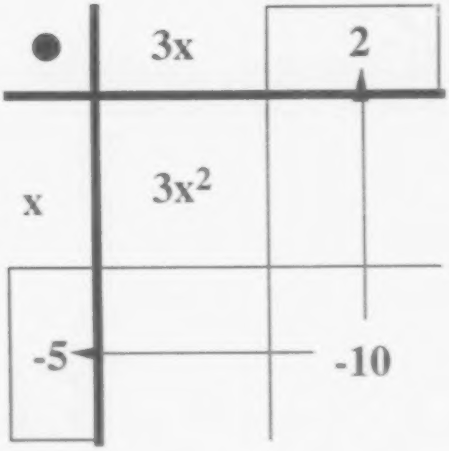
$$x^2 + (3x + 7x) + 21 =$$

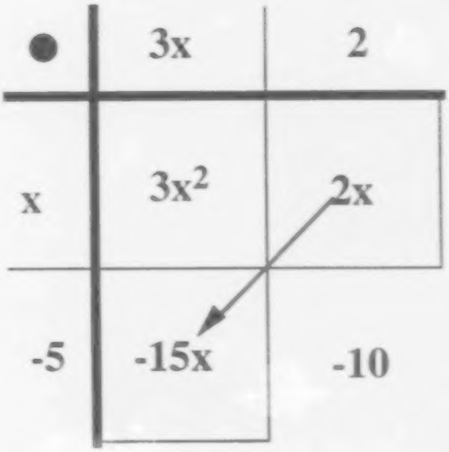
$$x^2 + 10x + 21$$

Factor $3x^2 - 13x - 10$.






●
3x
2
x
$3x^2$
-5
-10


●
3x
2
x
$3x^2$
2x
-5
-15x
-10

Since $-15x + 2x = -13x$ then $3x^2 - 13x - 10 = (3x + 2)(x - 5)$

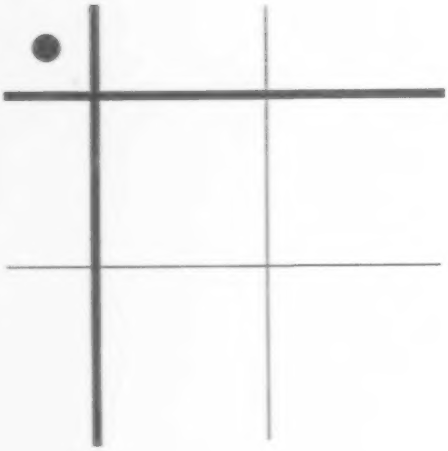
Multiplying Binomials

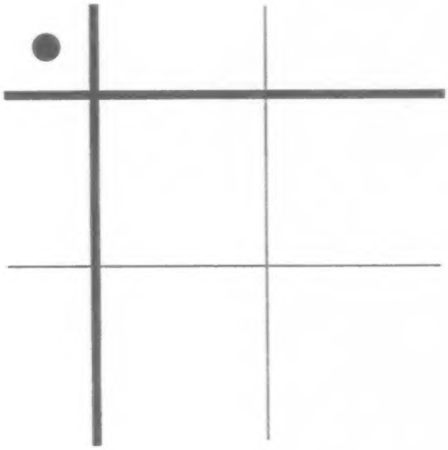
Fill in the space at the top of each column with the terms from one of the binomials. Fill in the space at the left side of the matrix for each row with the terms from the other binomial.

Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

In the matrix of the products, add along the diagonals from right to left to combine like terms and simplify.





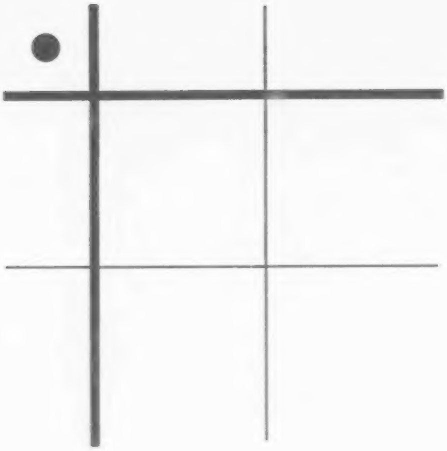
Factoring Quadratic Expressions

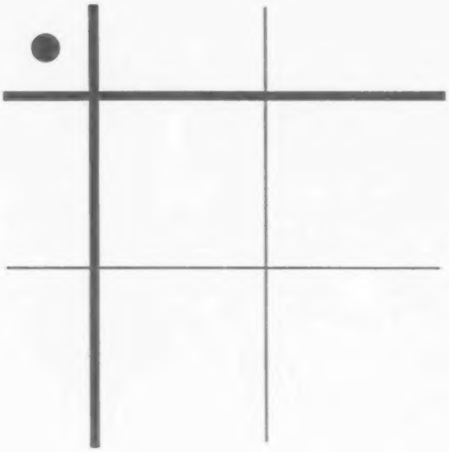
For a quadratic expression of the form $\mathbf{ax^2 + bx + c}$ (**b** can equal zero), place the $\mathbf{ax^2}$ term in the upper left location (first row, first column) and the **c** term in the lower right (second row, second column).

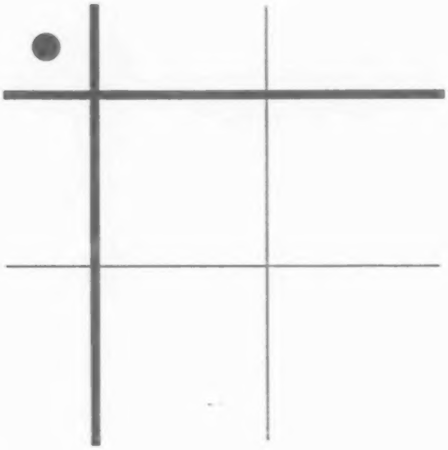
What are two possible factors of $\mathbf{ax^2}$? Place the factors in the locations along the boundary for the first row and the first column.

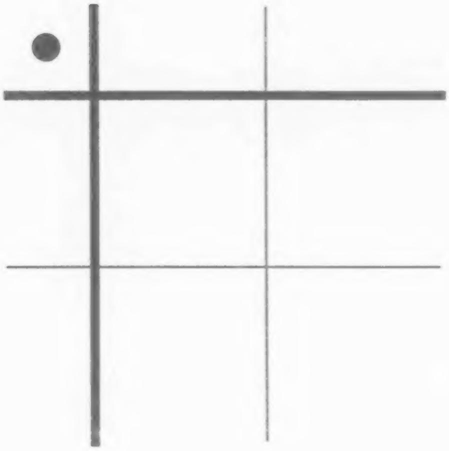
What are two possible factors of **c**? Place the factors in the locations along the boundary for the second row and the second column.

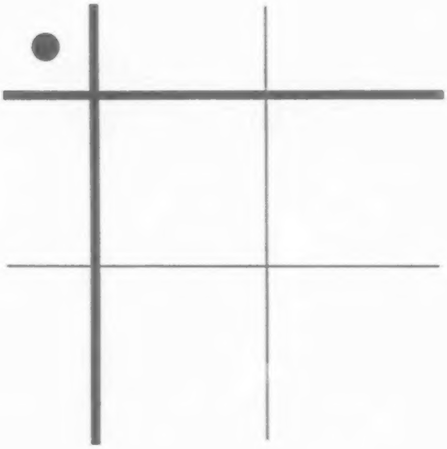
Multiply the binomials that are now in the boundary. Do the two diagonal products add up to **bx**? If not, go back and adjust the factors for **c** and/or $\mathbf{ax^2}$.

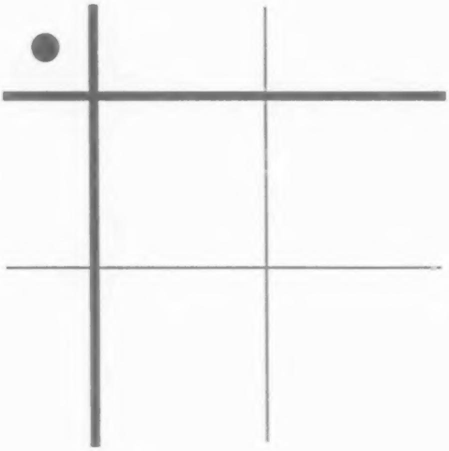












Polynomial Four in a Row

$x^2 - 4x + 4$	$2x^2 + x - 1$	x^2	$2x^2 + 5x + 3$	$x^2 + 4x + 4$	$x^2 - 1$
$x^2 - 2x + 1$	$4x^2 + 4x - 3$	$x^2 + 3x + 2$	$x^2 - 3x + 2$	$2x^2 - 5x + 2$	$2x^2 - x - 6$
$2x^2 - 3x + 1$	$x^2 - 2x$	$2x^2 + 3x$	$2x^2 + 5x + 2$	$4x^2 - 4x + 1$	$x^2 - 4$
$4x^2 + 4x + 1$	$2x^2 + 7x + 6$	$x^2 + 2x$	$2x^2 + 3x - 2$	$2x^2 - x - 1$	$2x^2 + x - 3$
$x^2 + 2x + 1$	$x^2 + x - 2$	$2x^2 - x$	$2x^2 + 3x + 1$	$4x^2 + 8x + 3$	$2x^2 - 3x - 2$
$x^2 - x - 2$	$2x^2 + x$	$4x^2 + 12x + 9$	$x^2 - x$	$4x^2 - 1$	$x^2 + x$

To play ***Polynomial Four in a Row***, you will need markers of two different shapes or colors and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly **ONE** clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally.

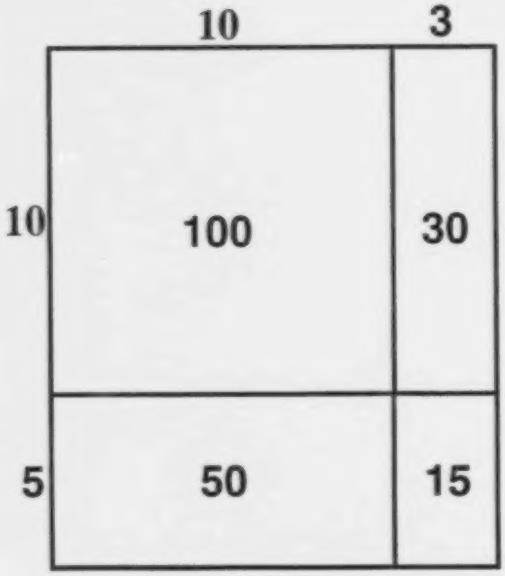
$2x - 1$	$x - 2$	$x - 1$	x	$x + 1$	$x + 2$	$2x + 1$	$2x + 3$

Four in a Row

To play ***Four in a Row***, you will need markers of two different shapes or colors and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly **ONE** clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally.

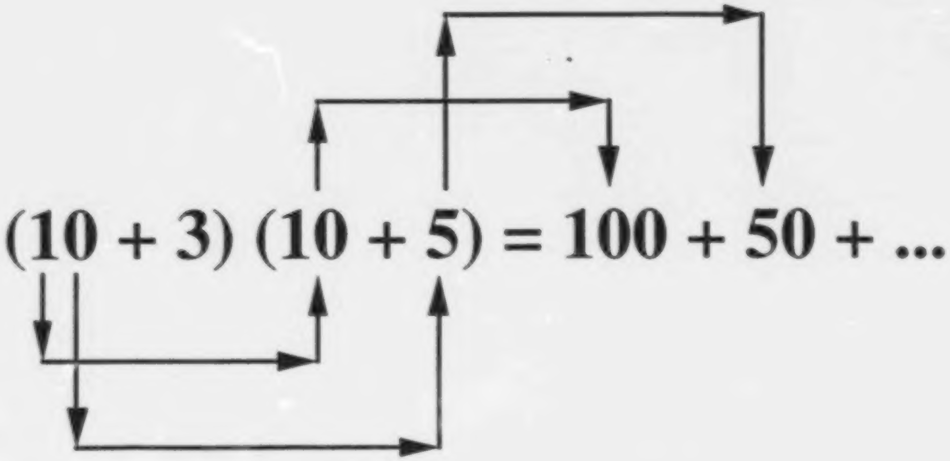
Area Models of Binomial Multiplication and the Distributive Property

How do you find the area of a rectangle that is 13m by 15 m?
Multiply 13 • 15.
Think of 13 • 15 = (10 + 3) (10 + 5).
If we divide the 13 by 15 rectangle into smaller rectangles like this, we have four smaller rectangles that together have the same area as the original rectangle.

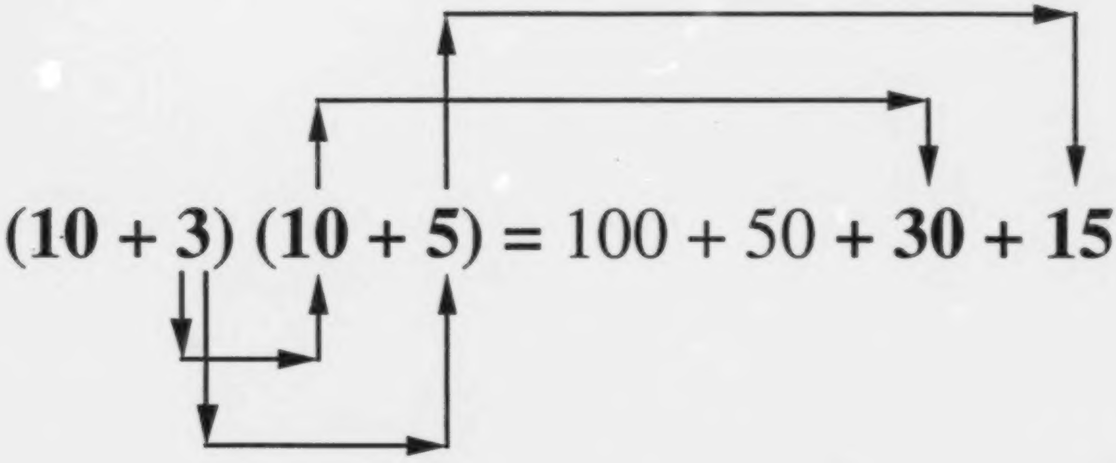


Notice how the areas of the smaller rectangles were found.
10 • 10 = 100, 10 • 5 = 50, 3 • 10 = 30, and 3 • 5 = 15.
In other words,
(10 + 3) (10 + 5) = 10 • 10 + 10 • 5 + 3 • 10 + 3 • 5 = 195 cm^2

Notice that the distributive property is applied twice.



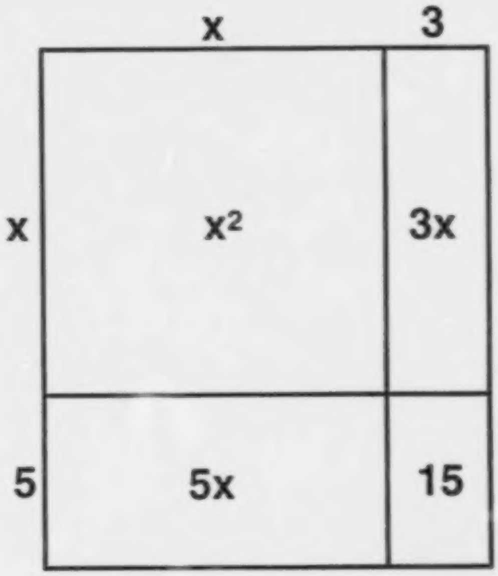
$$(10 + 3)(10 + 5) = 100 + 50 + \ldots$$



$$(10 + 3)(10 + 5) = 100 + 50 + \mathbf{30} + \mathbf{15}$$

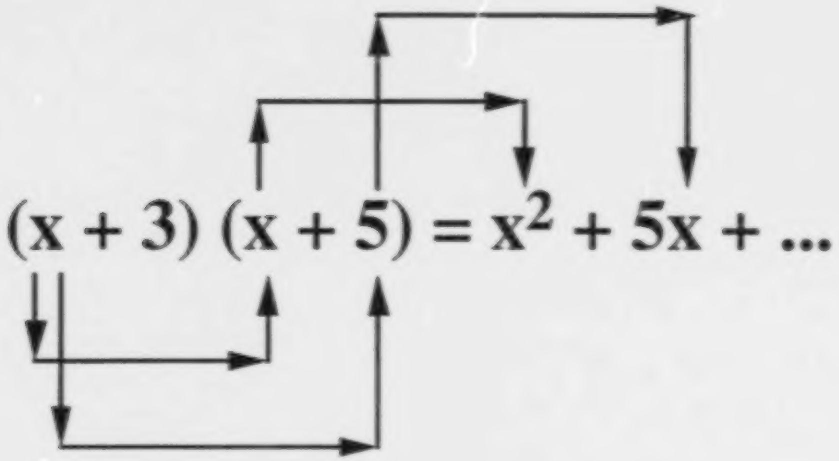
Area Models of Binomial Multiplication and the Distributive Property

How do you find the area of a rectangle that is (x + 3) by (x + 5)?
Multiply (x + 3) • (x + 5).
If we divide the (x + 3) by (x + 5) rectangle into smaller rectangles like this, we have four smaller rectangles that together have the same area as the original rectangle.

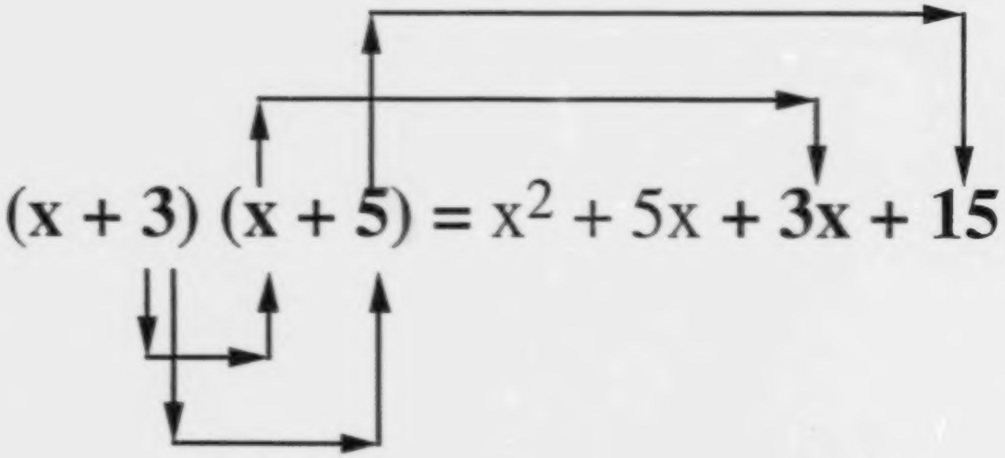


Notice how the areas of the smaller rectangles were found.
$x \cdot x = x^2$, $x \cdot 5 = 5x$, $3 \cdot x = 3x$, and $3 \cdot 5 = 15$.
In other words,
$(x + 3) \cdot (x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$.

Notice that the distributive property is applied twice.



$$(x + 3)\,(x + 5) = x^2 + 5x + \ldots$$



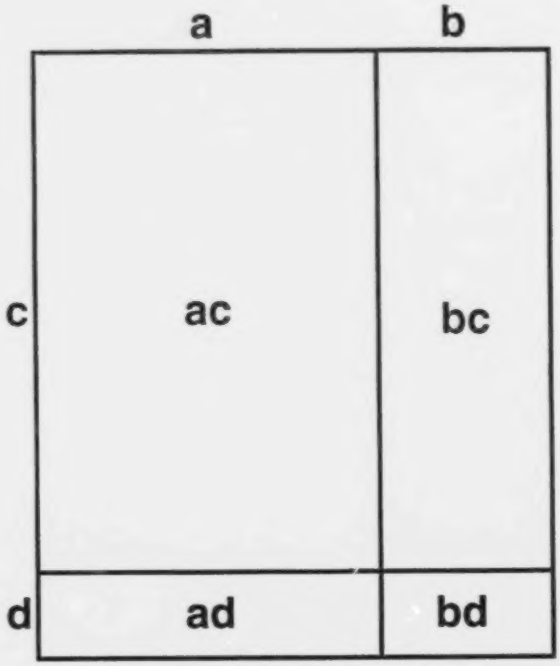
$$(x + 3)\,(x + 5) = x^2 + 5x + 3x + 15$$

Area Models of Binomial Multiplication and the Distributive Property

Finding the product of any two binomials $(a + b)$ and $(c + d)$ is just like finding the area of a $(a + b)$ by $(c + d)$ rectangle.

Multiply $(a + b) \bullet (c + d)$.

If we divide the $(a + b)$ by $(c + d)$ rectangle into smaller rectangles like this, we have four smaller rectangles that together have the same area as the original rectangle.



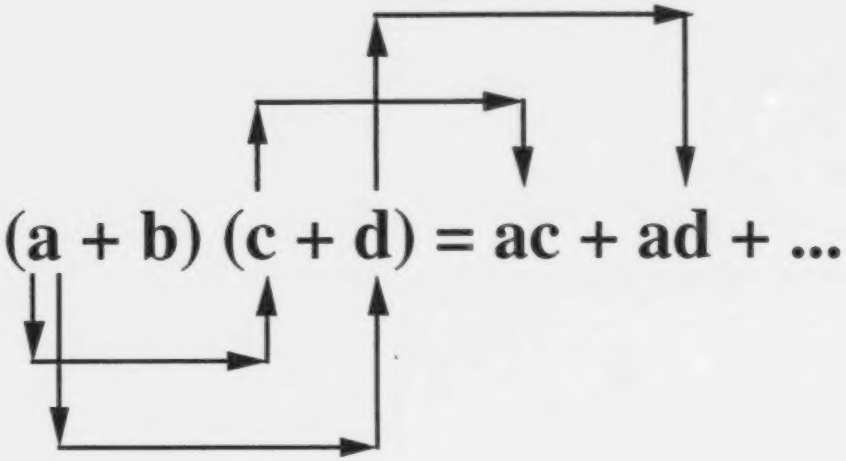
Notice how the areas of the smaller rectangles were found.

$a \bullet c = ac$, $a \bullet d = ad$, $b \bullet c = bc$, and $b \bullet d = bd$.

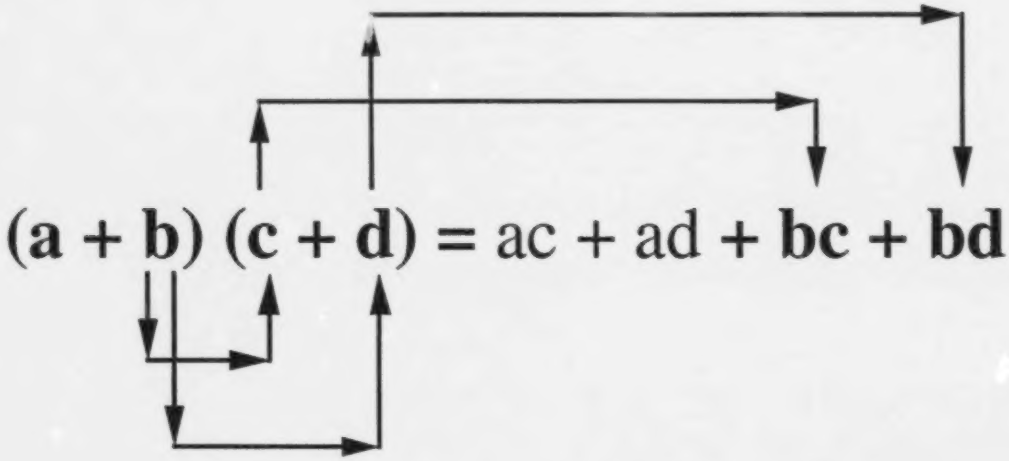
In other words,

$(a + b) \bullet (c + d) = ac + ad + bc + bd$

Notice that the distributive property is applied twice.



$$(a + b)(c + d) = ac + ad + \ldots$$

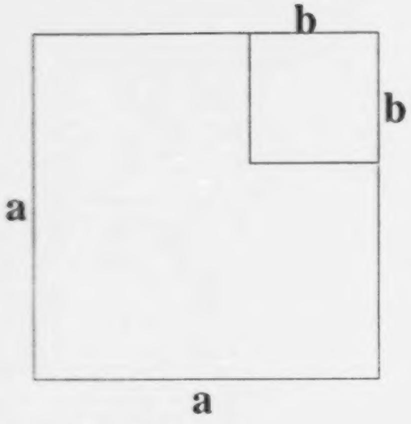


$$(a + b)(c + d) = ac + ad + \mathbf{bc} + \mathbf{bd}$$

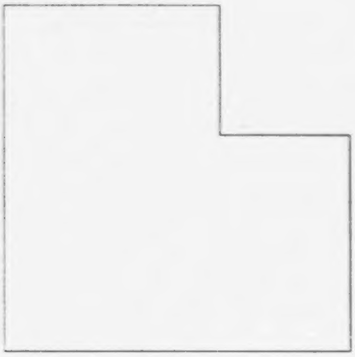
Algebra Uno

a	2a	4a	6a	8a	10a	12a	17
8b	6b	4b	3b	b	17a	14a	15
10b	13b	14b	17b	x	3x	5x	14
y	17x	15x	13x	10x	8x	6x	13
3y	5y	7y	8y	10y	13y	15y	12
10xy	9xy	7xy	5xy	3xy	xy	18y	10
13xy	15xy	18xy	2ab	3ab	5ab	7ab	9
$4x^2$	$2x^2$	18ab	15ab	13ab	12ab	9ab	8
$5x^2$	$7x^2$	$9x^2$	$12x^2$	$14x^2$	$15x^2$	$18x^2$	7
$14y^2$	$12y^2$	$9y^2$	$7y^2$	$6y^2$	$4y^2$	$2y^2$	6
$17y^2$	$18y^2$	$2a^2$	$4a^2$	$6a^2$	$8a^2$	$9a^2$	5
1	2	3	$20a^2$	$17a^2$	$14a^2$	$12a^2$	4

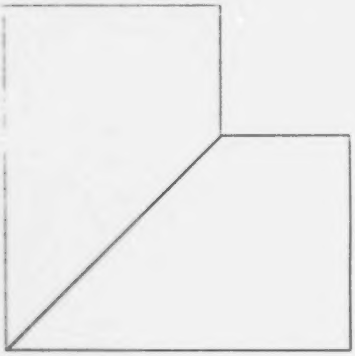
Difference of Two Squares, $a^2 - b^2$



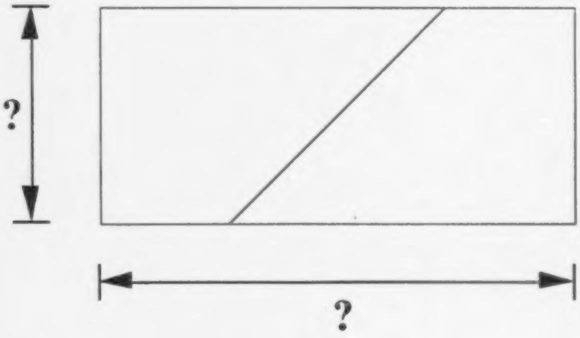
Cut a square with sides **a**. What is the area of the square? Draw a smaller square in the corner of the first square and label its sides **b**. What is the area of the smaller square?



Cut out the smaller square from the corner of the original square. What is the area of the new figure?



Cut the figure as shown. Rearrange the two pieces to form a rectangle. What is the area of the new rectangle?



What are the length and width of the rectangle?

Use the digits 0 - 9 to fill in the squares. Each digit can be used only once.

$$x^2 - x - \square = (x + 2)(x - \square)$$

$$x^2 - 1\square x + 1\square = (x - 2)(x - \square)$$

$$x^2 + \square x + 1\square = (x + \square)(x + 2)$$

$$x^2 - \square x - 24 = (x - 6)(x + \square)$$

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Make the Message

Simplify each expression.

Record the letter corresponding to the correct answer in the space provided.

A = $x - 1$

E = $x + 5$

H = $x - 7$

I = $x + 1$

T = $x + 2$

S = $x - 3$

Y = $x + 7$

(1) ____ $\dfrac{x^2 + 3x + 2}{x + 1}$

(2) ____ $\dfrac{x^2 - 49}{x + 7}$

(3) ____ $\dfrac{x^2 + 2x + 1}{x + 1}$

(4) ____ $\dfrac{3x^2 - 5x - 12}{3x + 4}$

(5) ____ $\dfrac{(x + 1)^2(x + 3)}{x^2 + 4x + 3}$

(6) ____ $\dfrac{3x^2 - 27}{3x + 9}$

(7) ____ $\dfrac{(x - 3)(x^2 + 6x + 5)}{x^2 - 2x - 3}$

(8) ____ $\dfrac{x^2 - 1}{x + 1}$

(9) ____ $\dfrac{2x^2 - 5x - 3}{2x + 1}$

(10) ____ $\dfrac{(x - 2)(x^2 + 8x + 7)}{x^2 - x - 2}$

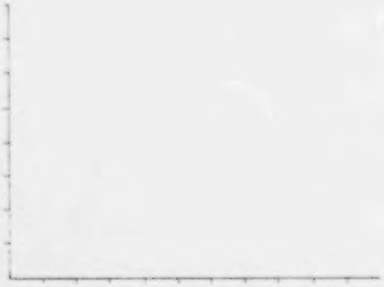
Drive Time

Tina's dad wishes to calculate the number of hours it will take to drive to certain vacation spots. He estimates that he can drive at an average speed of 58 mph.

1. Complete the chart.

Hours	Distance
2	
4	
6	
8	

2. Draw a graph of the relationship. What is the equation for the relationship?



3. Explain how to determine the distance he travels in 4.5 hours by using a proportion and by using a graph.

4. Explain how long it will take to travel 325 miles by using a proportion and by using a graph.

1996 NC Income Tax Schedule

Taxable Income		
is more than	but not over	Tax is
Single		
$ 0	$ 12,750	6% of the taxable income
12,750	60,000	$765 + 7% of the amount over $12,750
60,000	———	$4,072.50 + 7.75% of the amount over $60,000
Married filing jointly		
$ 0	$ 21,250	6% of the taxable amount
21,250	100,000	$1,275 + 7% of the amount over $21,250
100,000	———	$6,787.50 + 7.75% of the amount over $100,000
Married filing separately		
$ 0	$ 10,625	6% of the taxable amount
10,625	50,000	$637.50 + 7% of the amount over $10,625
50,000	———	$3,393.75 + 7.75% of the amount over $50,000

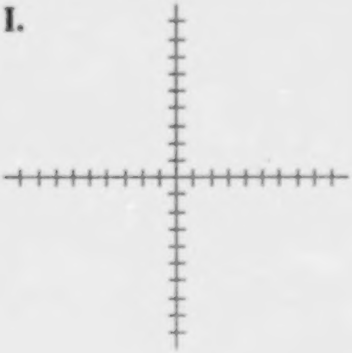
1996 Federal Tax Rate Schedule

Taxable Income		
is more than	but not over	Tax is
Single		
$ 0	$ 24,000	15% of the taxable income
24,000	58,150	$3,600 + 28% of the amount over $24,000
58,150	121,300	$13,162 + 31% of the amount over $58,150
121,300	263,750	$32,738.50 + 36% of the amount over $121,300
263,750	———	$84,020.50 + 39.6% of the amount over $263,750
Married, filing jointly		
$ 0	$ 40,100	15% of the taxable income
40,100	96,900	$6,015 + 28% of the amount over $40,100
96,900	147,700	$21,919 + 31% of the amount over $96,900
147,700	263,750	$37,667 + 36% of the amount over $147,700
263,750	———	$79,445 + 39.6% of the amount over $263,750
Married, filing separately		
$ 0	$ 20,050	15% of the taxable income
20,050	48,450	$3,007.50 + 28% of the amount over $20,050
48,450	73,850	$10,959.50 + 31% of the amount over $48,450
73,850	131,875	$18,833.50 + 36% of the amount over $73,850
131,875	———	$39,722.50 + 39.6% of the amount over $131,875

The Picture Tells the (Quadratic) Story

Use your graphing calculator to investigate each family of equations.
Sketch each equation's graph on the axes provided.
Answer the questions for each family of equations.

I.



$y = x^2$

$y = x^2 + 3$

$y = x^2 - 2$

- How are the curves alike?
- How are the curves different?
- Where does each curve cross the y-axis?
- What happens to the graph when a constant is added to **$y = x^2$**?
- Write equations for curves similar to those above but cross the y-axis at 5; at -7.

II.

$y = x^2$

$y = -x^2$

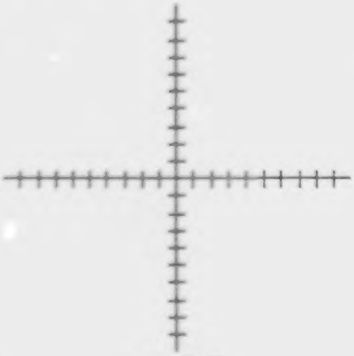
- How are the curves alike?
- How are the curves different?

The Picture Tells the (Quadratic) Story

III.

$y = x^2$

$y = 2x^2$



$y = 5x^2$

$y = -x^2$

$y = -2x^2$

$y = -3.5x^2$

$y = (1/2)x^2$

$y = (1/5)x^2$

$y = -0.4x^2$

- Describe the differences among the graphs.
- Which curve opens widest?
- What makes the difference?

Max-Min Problems

Suppose the formula **$y = 18x - 4x^2$** approximates (in yards) the height (**y**) that a kicked football will reach after **x** seconds.

1. Complete the following table:

seconds	height
0	
1	
2	
3	
4	
5	

2. Using your table, find the maximum height the football reaches and explain how you determined that height. How long does it take the football to reach that height?

3. Graph the equation on your calculator and find the same information that was requested in #2. What is the name of the location on the graph that provides this information?

Suppose Ralph's exam grade can be determined by the equation **$y = -2x^2 + 20x + 42$**, where **x** is the number of hours studied and **y** is the grade.

4. Complete the table and graph the equation.

hours	grade
0	
2	
4	
6	
8	
10	

5. According to this model, what is the highest grade Ralph can make on his exam? How many hours does he need to study to attain that grade?

6. What are the coordinates of the vertex of the graph of **$y = -2x^2 + 20x + 42$**? What does the vertex represent?

Finding a Best Fit Quadratic Equation

Enter the program into the calculator. **PRGM**. Go to the **EDIT** Menu. Choose an open entry for the program. Key in the name for the program. **ENTER**. Each line of the program is indicated by a colon (:). **PRGM**. Choose the **CTL** or **I/O** menus. Highlight (and **ENTER**) or key the number of the appropriate command. Also choose appropriate notation or commands from the keypad or other menus (look in **VARS** for example). **QUIT** after the last line.

```
ClrHome
LinReg
ClrHome
3->Arow
3->Acol
3->Brow
1->Bcol
Σ x²->[A](1,1)
Σ x²->[A](2,2)
Σ x²->[A](3,3)
Σ x->[A](1,2)
Σ x->[A](2,3)
n->[A](1,3)
Σ y->[B](1,1)
Σ xy->[B](2,1)
0->X
0->D
0->E
0->H
Lbl 1
X + 1->X
D+{x}(X)³->D
E+{x}(X)^4->E
H+{x}(X)²{y}(X)->H
If X=n
Goto 2
Goto 1
Lbl 2
D->[A](2,1)
D->[A](3,2)
E->[A](3,1)
H->[B](3,1)
[A]⁻¹[B]->[C]
"[C](1,1)X²+[C](2,1)X+[C](3,1)"->Y1
xSort
{x}(1)->Xmin
{x}(n)->Xmax
0->Xscl
ySort
{y}(1)->Ymin
{y}(n)->Ymax
0->Yscl
Scatter
0->Y
0->Z
Lbl 9
Z+1->Z
{x}(Z)->X
({y}(Z)-Y1)²+Y->Y
If Z<n
Goto 9
1-Y/(Σ y²-(Σ y)²/n)->W
√ W->W
Pause
Disp [C]
Disp W
```

- To run the program. Enter data.
- **PRGM**. Highlight (and **ENTER**) or key the number of the program to run. **ENTER**.
- The calculator will draw the scatter plot and the qraph of the best-fit quadratic equation.
- **ENTER**.
- The calculator will display the values for **a, b, c** and **r**. The program uses the general form of a quadratic equation, $\mathbf{y = ax^2 + bx + c}$. **r** is the correlation coefficient.

US Health Expenditures

(billions of dollars)

1960	27.1
1970	74.4
1980	250.1
1985	422.6
1986	454.8
1987	494.1
1988	546.0
1989	602.8
1990	666.2

Federal Budget Projections

(billions of dollars)

Year	Revenue	Expenditures
1995	1355	1531
1996	1418	1625
1997	1475	1699
1998	1546	1769
1999	1618	1872
2000	1697	1981
2001	1787	2084
2002	1880	2202

Rolling Dice: An Exponential Experience

Place 30+ dice in a cup and roll them. Remove the 3s and record the results in the table below.

Roll the remaining dice and again remove the 3s. Record you results.

Continue to roll and remove 3s until you have only one or two dice left. Keep accurate records of your results.

Roll	Remaing Dice	Roll	Remaining Dice	Roll	Remaining Dice
1		10		19	
2		11		20	
3		12		21	
4		13		22	
5		14		23	
6		15		24	
7		16		25	
8		17		26	
9		18		27	

Continue on the back of your paper if necessary.

Use your calculator to plot the data. Which quantity is independent (**x**) and which is dependent (**y**)? Is the function increasing or decreasing?

Using your calculator, find the best fit exponential equation.

What was the correlation coefficient? What does that mean?

How does your best fit equation compare with $\mathbf{Y = N \cdot (5/6)^x}$ (N is the number of dice you began with)?

Fill in each table of values according to the equations provided.

$y = x + 3$

x	y
1	
2	
3	
4	
5	

$y = 3x$

x	y
1	
2	
3	
4	
5	

$y = x^3$

x	y
1	
2	
3	
4	
5	

$y = 3^x$

x	y
1	
2	
3	
4	
5	

What patterns do you notice in the data?
What happens when the variable is an exponent?
Graph the equations and compare the graphs.

Which table of values represents an exponential relationship?

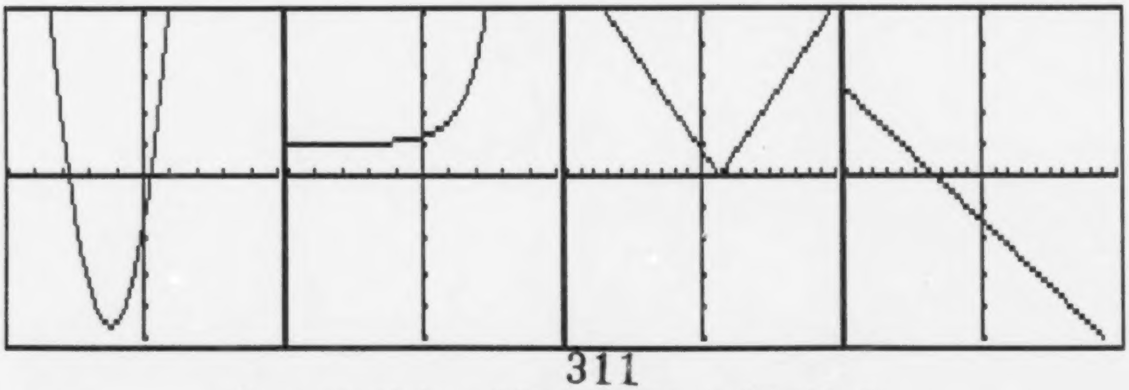
x	y
0	32
4	48
8	72
12	108
16	162

x	y
0	35
4	199
8	523
12	1007
16	1651

x	y
0	30
4	50
8	70
12	90
16	110

x	y
0	31
4	93
8	279
12	837
16	2511

Which graph represents an exponential relationship?



Use Your Imagination

The Sears Tower in Chicago is 1,454 feet high. Suppose you take a sheet of paper and fold it once in half, then fold it in half again, and keep folding it over and over again. How many times would you have to fold it to create a piece of paper that is higher than the Sears Tower?

Standing at sea level, what would be the fewest number of times you need to fold your paper to exceed the height (29,028 feet) of Mount Everest?

You go to NASA and tell them you have a better way of returning to the Moon. "I can fold this paper and reach the Moon. Watch." How many times do you need to fold the paper?

The Dinosaurs Bite the Dust

Names ______________________________ **Date** __________

______________________________ **Period** __________

The 50 dice provided represent the population of large dinosaurs (500,000).

Place the dice in a cup and roll them. Remove all the dice that show 5. These 5s represent the dinosaurs which died during the first year after the extinction event. Record the number of dinosaurs (dice) **remaining**.

Roll the remaining dice and again remove the 5s. Each roll represents another year since the extinction event. Continue the process until there is only one or two dice remaining.

Keep a record of the results for each roll.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________
21. ________
22. ________
23. ________
24. ________
25. ________
26. ________
27. ________
28. ________
29. ________
30. ________
31. ________
32. ________

Is the data linear? exponential? ________________________

Use the calculator to determine best-fit linear and exponential functions.

linear function ____________________ exponential function ____________________

Which fits best? ______________ Why? ______________________________________

How do the best-fit functions compare with the expected function $\mathbf{y = N \cdot (5/6)^x}$, where **N** is the number of dice you begin with, **x** is the number of rolls, and **y** is remaining dice?

Compile all of the data collected by the class, graph the data, and determine a best-fit exponential function. How does the best-fit function compare with the expected function above?

How long after the extinction began are there fewer than 100 large dinosaurs?

Problems of an Exponential Nature

Jerry has joined the Peace Corps and is studying Malagasy at the language school. Because he entered the school late, he was only able to learn 100 key phrases before the month long Christmas recess. Jerry returns to his hometown for the holiday and does not practice. According to language experts, without continual practice or immersion, a person will forget 0.5% daily of any new language they are studying. How many of the key phrases does Jerry remember the day before he returns to the language school in January? Back at school, it takes Jerry just a short time to recall the forgotten phrases.

At age 22, you are offered a contract for a starting salary of $22,500 with a $2,000 increase in salary each year OR $22,500 with a 5% increase in salary per year.

Complete the table for each offer and graph to compare the differences.

Years	Salary + $2000
0	
10	
20	
30	
40	

Years	Salary + 5%
0	
10	
20	
30	
40	

If you plan to work for this company for 30 years, which contract would you choose? Which would you choose if you know you will work for this company only about six years?

Lining Up Dominoes

BEGIN	4(-2)	-8	(-1)(5)	-5	5(2)	10	(-3)(-4)
12	(2)(-3)	-6	0(3)	0	(1)(-1)	-1	(2)(-1)
-2	(-4)(1)	-4	(-1)(-5)	5	(-3)(-1)	3	(2)(2)
4	(2)(3)	6	(-4)(-2)	8	(3)(3)	9	(-3)(3)
-9	(-2)(5)	-10	(-2)(6)	-12	2(-8)	-16	(-4)(-4)
16	(1)(2)	2	5(-3)	-15	3(5)	15	(-3)(-6)
18	(-2)(9)	-18	(-4)(25)	-100	10(-2)	-20	(5)(5)
25	(-10)(-10)	100	4(5)	20	5(-5)	-25	END

Lining Up Dominoes

START	$2(x-3)$	$2x-6$	$-9(8x-y)$	$-72x+9y$	$-(3x-1)$	$-3x+1$	$9(8x-y)$
$72x-9y$	$-2(x-3)$	$-2x+6$	$-12(-x-2y)$	$12+24y$	$-4(x+6)$	$-4x-24$	$2(x+6)$
$2x+12$	$4(x+6)$	$4x+24$	$-5(2y+3x)$	$-15x-10y$	$-(3x+1)$	$-3x-1$	$12(x-2y)$
$12x-24y$	$2(x+3)$	$2x+6$	$-3(4x-y)$	$-12x+3y$	$-2(x+3)$	$-2x-6$	$-7(8x-9y)$
$-56x+63y$	$-4(x-6)$	$-4x+24$	$3(4x+y)$	$12x+3y$	$-8(4-x)$	$8x-32$	$-12(x+2y)$
$-12x-24y$	$6(2x+6)$	$12x+36$	$5(2y-3x)$	$-15x+10y$	$4(x-6)$	$4x-24$	$-12(x-2y)$
$-12x+24y$	$7(8x+9y)$	$56x+63y$	$12(3x-1)$	$36x-12$	$8(4-x)$	$-8x+32$	$3(4x-y)$
$12x-3y$	$2(x-6)$	$2x-12$	$4(x-3)$	$4x-12$	$-9(-8x-y)$	$72x+9y$	STOP

Lining Up Dominoes
Master Sheet

I Have ... Who Has ...

I Have x	Who Has my expression multiplied by 2	I Have x - 2	Who Has my expression increased by 6
I Have 2x	Who Has 3 less than my expression	I Have x + 4	Who Has my expression multiplied by 4
I Have 2x - 3	Who Has my expression decreased by x	I Have 4x + 16	Who Has my expression decreased by 3x
I Have x - 3	Who Has my expression squared	I Have x + 16	Who Has my expression added with -10
I Have $x^2 - 6x + 9$	Who Has my expression added with -9	I Have x + 6	Who Has my expression multiplied by x - 6
I Have $x^2 - 6x$	Who Has my expression divided by x	I Have $x^2 - 36$	Who Has my expression decreased by 5x
I Have x - 6	Who Has my expression added with x^2	I Have $x^2 - 5x - 36$	Who Has my expression divided by x - 9
I Have $x^2 + x - 6$	Who Has my expression divided by x + 3	I Have x + 4	Who Has my expression multiplied by 2x

I Have ... Who Has ...

I Have $2x^2 + 8x$	Who Has my expression increased by 10	I Have $2x$	Who Has my expression squared
I Have $2x^2 + 8x + 10$	Who Has my expression divided by 2	I Have $4x^2$	Who Has my expression decreased by $3x^2$
I Have $x^2 + 4x + 5$	Who Has my expression decreased by $8x$	I Have x^2	Who Has my expression decreased by 100
I Have $x^2 - 4x + 5$	Who Has my expression added with -10	I Have $x^2 - 100$	Who Has my expression divided by $x + 10$
I Have $x^2 - 4x - 5$	Who Has my expression divided by $x - 5$	I Have $x - 10$	Who Has my expression added with $x + 12$
I Have $x + 1$	Who Has my expression added with $3x + 5$	I Have $2x + 2$	Who Has my expression divided by 2
I Have $4x + 6$	Who Has my expression divided by 2	I Have $x + 1$	Who Has 1 less than my expression
I Have $2x + 3$	Who Has my expression added with -3	I Have	Who Has

I Have ... Who Has ...

I Have -10	Who has my number increased by 3	I Have -7	Who has my number decreased by 5
I Have -12	Who has my number plus -8	I Have -20	Who has my number minus -4
I Have -16	Who has my number plus 20	I Have 4	Who has my number plus -30
I Have -26	Who has my number plus -2	I Have -28	Who has my number minus -13
I Have -15	Who has my number minus 8	I Have -23	Who has my number plus 30
I Have 7	Who has my number plus -5	I Have 2	Who has my number plus -21
I Have -19	Who has my number minus 8	I Have -27	Who has my number minus -13
I Have -14	Who has my number plus -16	I Have -30	Who has my number increased by 30

I Have 0	Who has my number decreased by 50	I Have -50	Who has my number increased by 60
I Have 10	Who has my number increased by 15	I Have 25	Who has my number plus -20
I Have 5	Who has my number plus 17	I Have 22	Who has my number plus 22
I Have 44	Who has my number minus -6	I Have 50	Who has my number plus 10
I Have 60	Who has my number decreased by 20	I Have 40	Who has my number plus -14
I Have 26	Who has my number minus 27	I Have -1	Who has my number plus 13
I Have 12	Who has my number plus -22	I Have	Who Has
I Have	Who Has	I Have	Who Has

I Have ... Who Has ...

I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has
I Have	Who Has	I Have	Who Has

Scientifico Gameboards

Between 1 and 50	Between 51 and 100	Between 101 and 500	Between 501 and 1000
Between 1001 and 5000	Between 5001 and 10,000	Between 10,001 and 50,000	Between 50,001 and 100,000
Between 100,001 and 500,000	Between 500,001 and 1,000,000	Between 1,000,001 and 5,000,000	Between 5,000,001 and 10,000,000

.000001 to .0000049	.000005 to .000009	.00001 to .000049	.00005 to .00009
.0001 to .00049	.0005 to .0009	.001 to .0049	.005 to .009
.01 to .049	.05 to .09	.1 to .49	.5 to .9

Scoring and Winning: 1996 NFL

	Record (wins-loses)	**Offense** (points scored)	**Defense** (points allowed)
Arizona	7-9	300	397
Atlanta	3-13	309	465
Baltimore	4-12	371	441
Buffalo	10-6	319	266
Carolina	12-4	367	218
Chicago	7-9	283	305
Cincinnati	8-8	372	369
Dallas	10-6	286	250
Denver	13-3	391	275
Detroit	5-11	302	368
Green Bay	13-3	456	210
Houston	8-8	345	319
Indianapolis	9-7	317	334
Jacksonville	9-7	325	335
Kansas City	9-7	297	300
Miami	8-8	339	325
Minnesota	9-7	298	315
New England	11-5	418	313
New Orleans	3-13	229	339
NY Giants	6-10	242	297
NY Jets	1-15	279	454
Oakland	7-9	340	293
Philadelphia	10-6	363	341
Pittsburgh	10-6	344	257
St. Louis	6-10	303	409
San Diego	8-8	310	376
San Francisco	12-4	398	257
Seattle	7-9	317	376
Tampa Bay	6-10	221	293
Washington	9-7	364	312

Gulliver's Clothes: Recording Sheet

Name	Thumb	Wrist	Neck	(Thumb,Wrist)	(Wrist/Neck)

How Do They Fit?

n = 15 n + 7 = -11 n = -20 n + 4 = 7	n = 31 n + 7 = -13 n = 23 n = -48	n - 48 = -17 n - 38 = -15 n - 5 = -30 n - 17 = 3
n = 3 n = -34 n - 26 = -8 n - 8 = -15	n + 12 = -36 n = 18 n = -39 n - 9 = 11	n = 20 n + 17 = -22 n + 25 = -23 n - 5 = -16
n = -7 n = 18 n = -25 n - 7 = -14	n = 20 n + 13 = -12 n - 5 = 9 n + 19 = -5	n = -11 n = 14 n + 5 = 8 n = -39

How Do They Fit?

$y < -4$ $y > 1/2$ $y < -1$ $y + 1 > -2$	$y + 2 < 8$ $-7y > 7$ $y > -4$ $12y < -6$	$y > -6$ $y - 8 > -12$ $y < -3$ $y/-3 > 5/-3$
$y > -3$ $y/2 > -1$ $y > 6$ $y/2 < 2$	$y < -1/2$ $y/-2 < -3$ $y < 3$ $7y < -14$	$y < 5$ $-2y > -6$ $15y < 3$ $y + 3 > 5$
$y < 4$ $y < 2$ $y > 1/3$ $5y > 15$	$y < -2$ $-6y < -2$ $y > 1$ $y > -1$	$y > 2$ $y - 8 > -7$ $y > 4$ $y - 4 < -3$

How Do They Fit?

$2x^2 - x - 1$ $(x - 3)(2x + 1)$ $x^2 - 25$ $(3x + 2y)^2$	$(2x + 1)(x - 1)$ $(x + 5)(x - 5)$ $(x + 3)^2$ $25x^2 + 30xy + 9y^2$	$(2x - 5)^2$ $x^2 + 6x + 9$ $x^2 + 9$ $(6x^2 - 4)^2$
$9x^2 + 12xy + 4y^2$ $x^2 + 3m - 18$ $(3x - 2y)(3x + 2y)$ $(x + 5)^2$	$(5x + 3y)^2$ $9x^2 - 4y^2$ $x^2 + 3x - 18$ $(x + 3)(x - 3)$	$36x^4 - 48x^2 + 16$ $(x - 3)(x + 6)$ $36x^4 + 48x^2 - 16$ $4x^2 - 25$
$x^2 + 10x + 25$ $9x^2 + 4y^2$ $(x + 3)(x - 2)$ $(x - 4)^2$	$x^2 - 9$ $x^2 + x - 6$ $x^2 - 3x - 18$ $x^2 + 6x - 6$	$(2x + 5)(2x - 5)$ $(x + 3)(x - 6)$ $x^2 - 6x + 9$ $(x + 1)(x - 6)$

How Do They Fit?

-3 $40 - 5\sqrt{6} + 8\sqrt{2} - 2\sqrt{3}$ $-\sqrt{6}$ $72 - 8\sqrt{5} + 9\sqrt{7} - \sqrt{35}$	$10 + 5\sqrt{5} + 2\sqrt{2} + \sqrt{10}$ $(2 + \sqrt{2})(4 + \sqrt{2})$ $(1 - \sqrt{3})(1 + \sqrt{3})$ $(8 - \sqrt{6})(5 + \sqrt{2})$	$(9 - \sqrt{5})(8 + \sqrt{7})$ $10 + 5\sqrt{5} + 2\sqrt{8} + 2\sqrt{10}$ $(2 + \sqrt{7})(2 - \sqrt{7})$ $18 + 8\sqrt{2}$
$(2 - \sqrt{6})(3 + \sqrt{6})$ $(4 + \sqrt{3})(8 + \sqrt{3})$ $14 + 14\sqrt{2} - 2\sqrt{6} - 4\sqrt{3}$ $(\sqrt{2} - \sqrt{5})(\sqrt{2} + \sqrt{5})$	-2 $(\sqrt{6} + \sqrt{8})(3 - \sqrt{8})$ $(3 + \sqrt{3})(3 - \sqrt{3})$ $35 + 12\sqrt{3}$	-3 $(2 + \sqrt{5})(5 + \sqrt{2})$ $(4 - \sqrt{3})(3 + \sqrt{3})$ $3\sqrt{6} + 3\sqrt{8} - 4\sqrt{3} - 8$
$(2 + \sqrt{8})(7 - \sqrt{6})$ $(4 + \sqrt{2})(3 - \sqrt{2})$ $(3 - \sqrt{2})(2 + \sqrt{3})$ $4\sqrt{3} + 4 - \sqrt{30} - \sqrt{10}$	6 $(7 - \sqrt{3})(6 + \sqrt{10})$ $(2 + \sqrt{5})(5 + \sqrt{8})$ $10 - \sqrt{2}$	$9 + \sqrt{3}$ $6 - 2\sqrt{2} + 3\sqrt{3} - \sqrt{6}$ $(\sqrt{6} + \sqrt{2})(\sqrt{8} - \sqrt{5})$ $42 - 6\sqrt{3} + 7\sqrt{10} - \sqrt{30}$

How Do They Fit?
Master Sheet

Equation Relays

1.) $2x + 3x = 25$

2.) $5m - 8m = 36$

3.) $4(7x - 3) = 16$

4.) $25 = 5(2t - 7)$

5.) $10n - 8 + 4n = 20$

6.) $4a + 8 - a = 80$

7.) $15r - 18r = 687$

8.) $3(m + 1) + 2m = 88$

9.) $7(3k + 4) - 18k = 64$

10.) $10 = 10(y - 5) + 5y$

11.)

12.)

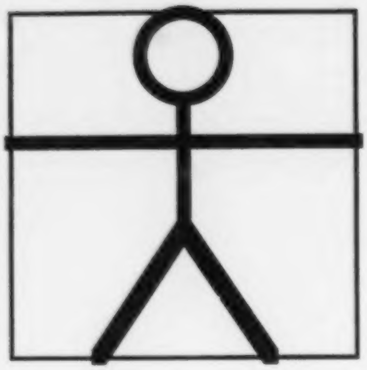
13.)

14.)

15.)

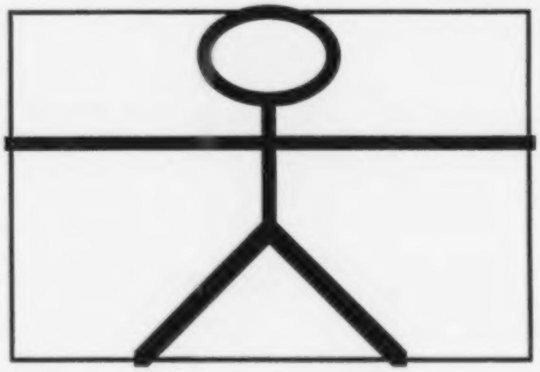
16.)

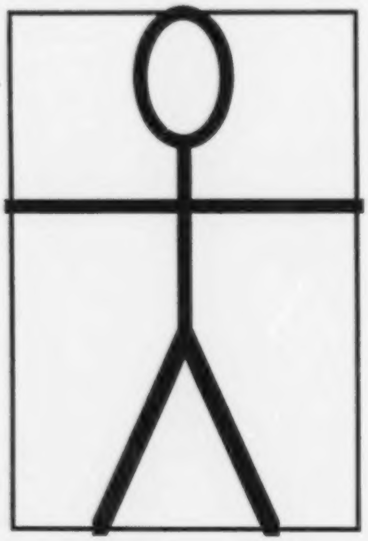
What Shape Are You?



Square
height = arm span

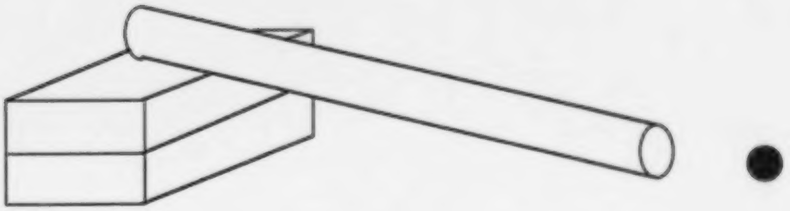
Short Rectangular
height < arm span





Tall Rectangular
height > arm span

It's All Downhill From Here



Follow the procedure for 3 trials at each height.

1. Roll the ball down the ramp. (Remember to release the ball, not push it.)
2. Measure and record the actual length of the roll.
3. Using the data collected from the 3 trials on the previous ramp, estimate the distance for the next ramp.

Ramp Height	Trial	Estimated Distance	Actual Distance
1	1		
	2		
	3		
2	1		
	2		
	3		
3	1		
	2		
	3	338	

Connecting Units of Measure

Recording Sheet

Team ______________________________

Object

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

____________________ __________cm __________in

Calculations

Point A ___________

Difference of centimeter values __________

Point B ___________

Difference of inch values ___________

Slope of the line __________

Percent of error ___________

Conclusion:

____

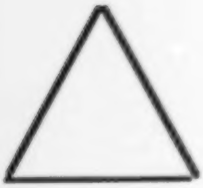
____

____

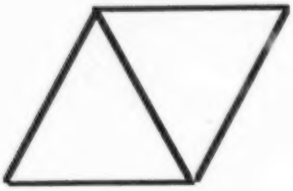
Did you enjoy the lab? ______ Yes ______ No

Did each member of your group contribute? ______ Yes ______ No

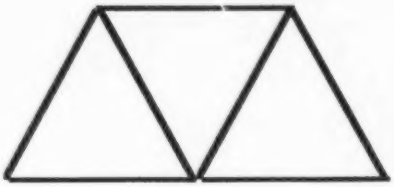
Toothpick Triangles



1



2



3

Use the pattern shown above.

1. Make the next three shapes.

2. Determine how many toothpicks are needed to make the 5th and 6th shapes in the pattern.

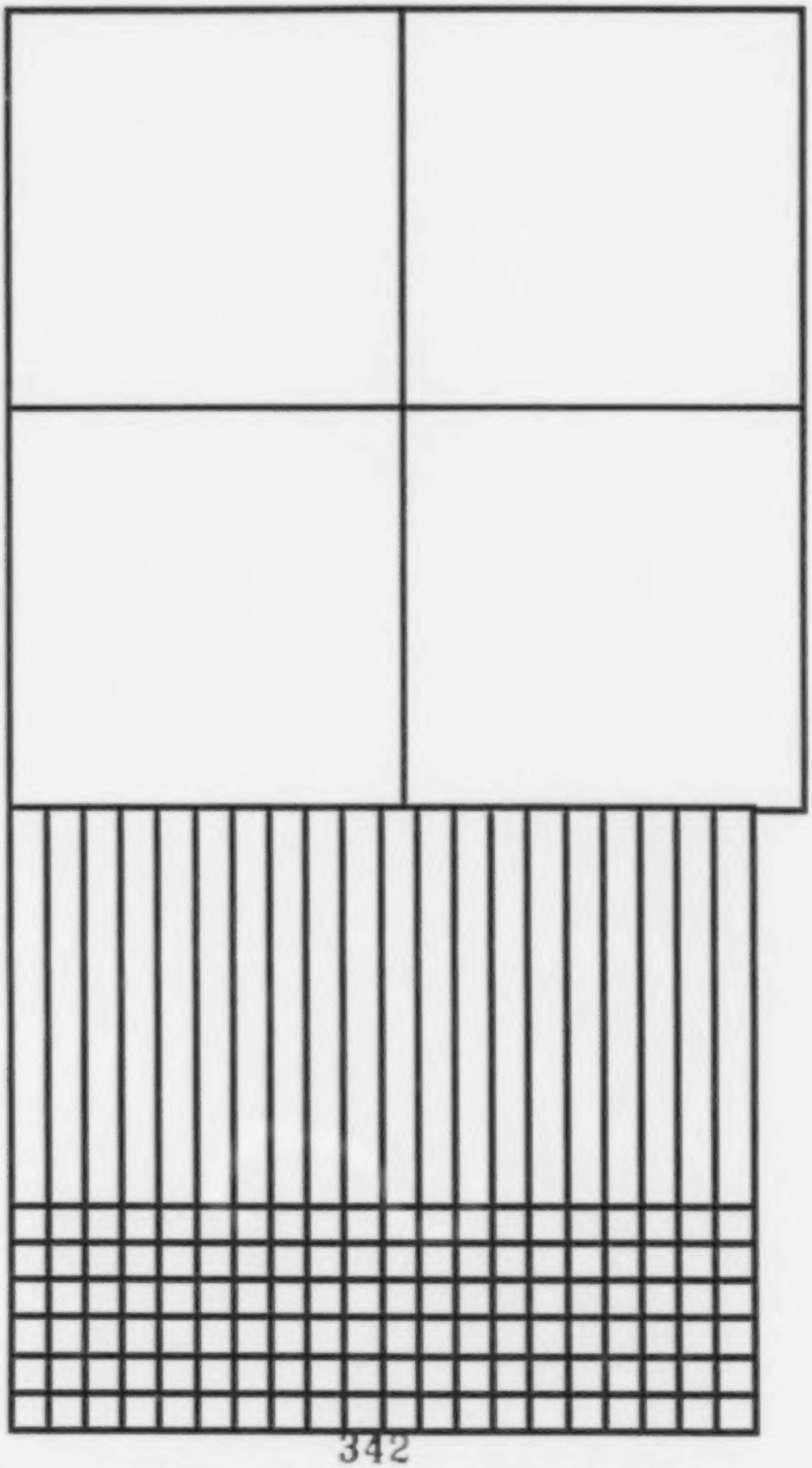
3. How many toothpicks are needed for the 11th shape? 50th? 500th?

4. State how the pattern grows.

Triangles	Toothpicks
1	3
2	5
3	
4	
n	

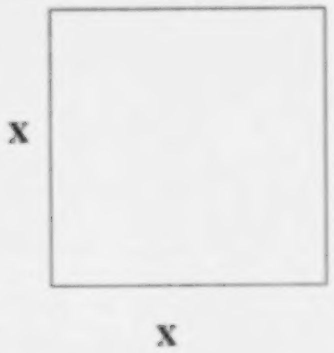
Algebra				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

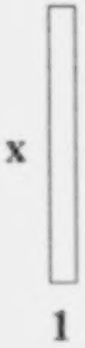
Algebra Tiles

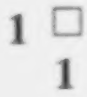


Multiplication with Algebra Tiles

What is the area of each figure?







Area = __________ Area = __________ Area = __________

Make rectangles using algebra tiles to model the following products. Sketch a picture of your models.

1. $(x + 3)(x + 7) =$ ____________________

2. $x(x + 5) =$ ____________________

3. $(x + 4)(x + 2) =$ ____________________

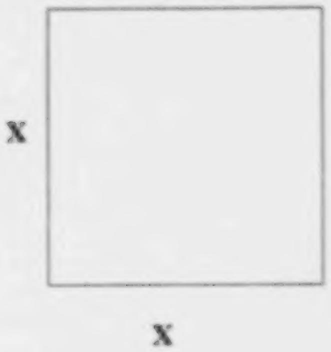
4. $(x + 3)(x + 4) =$ ____________________

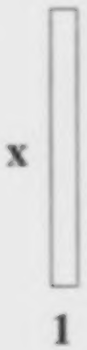
5. $(2x + 1)(x + 2) =$ ____________________

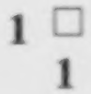
6. $(2x + 3)(2x + 1) =$ ____________________

Factoring with Algebra Tiles

What is the area of each figure?

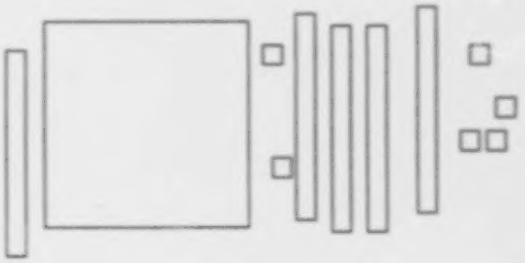






Area = __________ Area = __________ Area = __________

What is the algebraic expression represented by these tiles? __________________________



Using the same number and type of tiles at your desk, construct a rectangle (no "holes" allowed).

What are the dimensions of your rectangle? ____

For each of the following: make a rectangle with the given area,
draw your result,
and find the factors of the polynomials.

1. $x^2 + 7x + 10 =$ ____________________

2. $x^2 + 6x + 5 =$ ____________________

3. $x^2 + 6x + 9 =$ ____________________

4. $x^2 + 8x + 15 =$ ____________________

5. $x^2 + 7x + 12 =$ ____________________

6. $2x^2 + 11x + 12 =$ ____________________

7. $2x^2 + 7x + 3 =$ ____________________

8. $3x^2 + 5x + 2 =$ ____________________

9. $2x^2 + 7x + 6 =$ ____________________

10. $2x^2 + 9x + 4 =$ ____________________

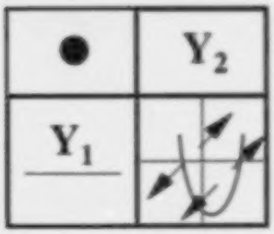
11. $2x^2 + 5x + 3 =$ ____________________

12. $4x^2 + 9x + 2 =$ ____________________

13. $2x^2 + 11x + 5 =$ ____________________

14. $3x^2 + 10x + 8 =$ ____________________

	$x + 2$		$x + 5$		$x + 3$		$3x - 7$	
$x - 5$								
$x + 3$								
$2x + 4$								



Operating With Binomials
Recording Sheet

In the table below:
Find the product of Y_1 and Y_2 algebraically.
Solve for x when $Y_1 \cdot Y_2 = 0$.

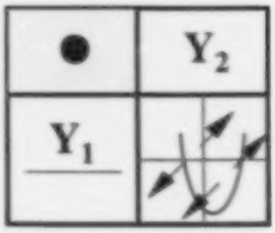
Y_1	Y_2	$Y_1 \cdot Y_2$	When $Y_1 \cdot Y_2 = 0$, x = ???
x - 5	x + 2		

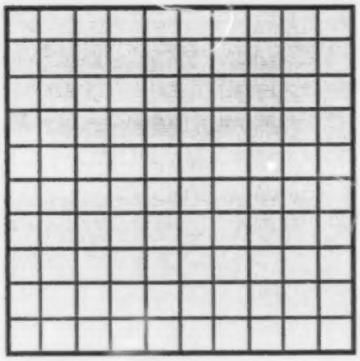
Generally, what is the product of two binomials called?

Compare the algebraic solutions in the last column of the table with the corresponding graphs on the other sheet. Explain.

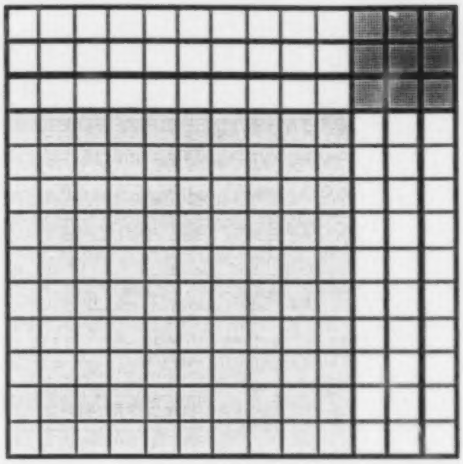
What are the other names for an equation's solution(s)?

When is the solution for an equation important? Name an example that is NOT strictly mathematical.

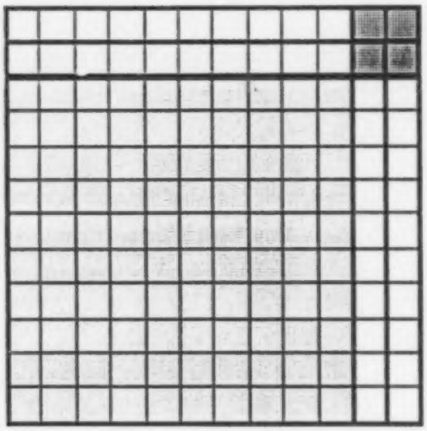




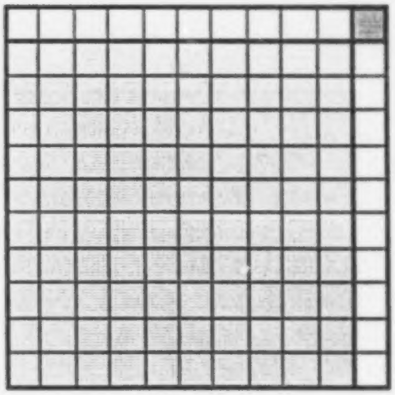
A



B

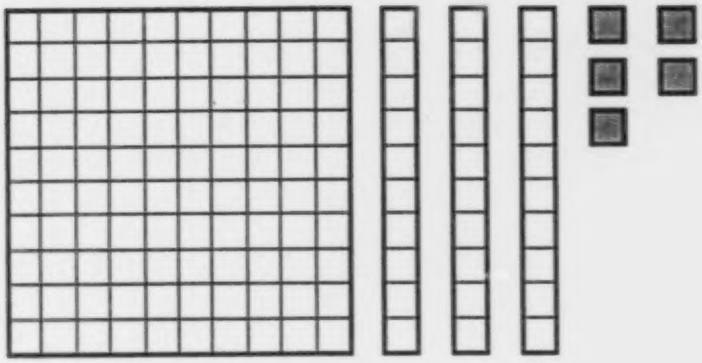


C

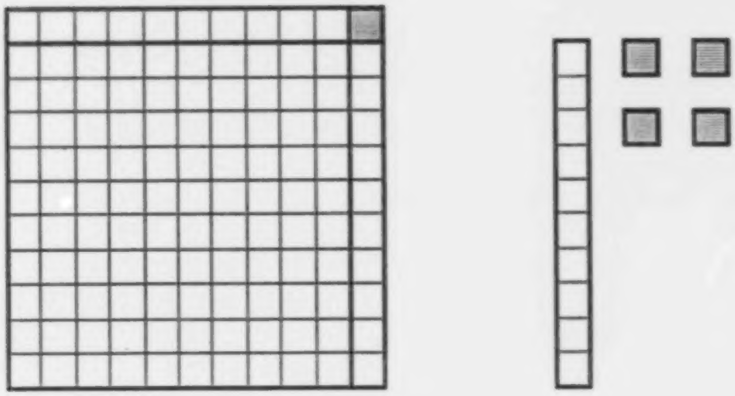


D

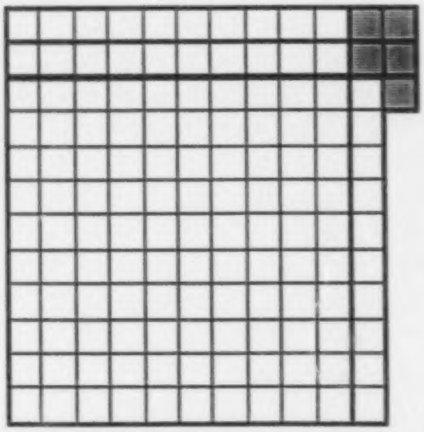
A



B



C



Getting to the Root of the Number

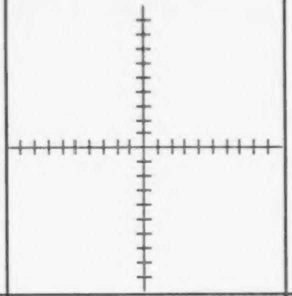
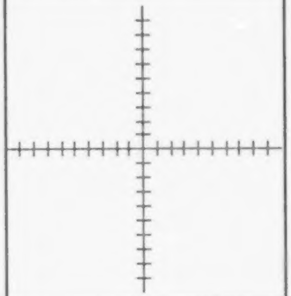
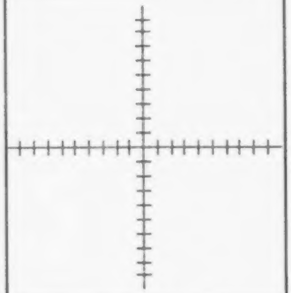
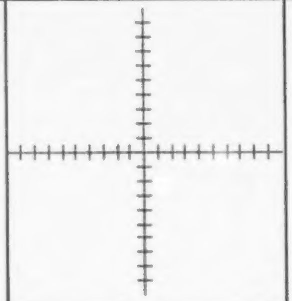
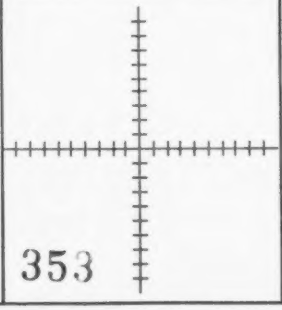
Names ____________________ Date ____________________

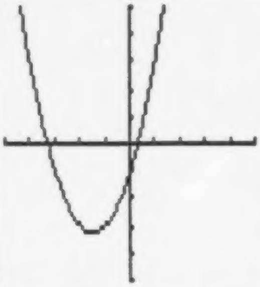
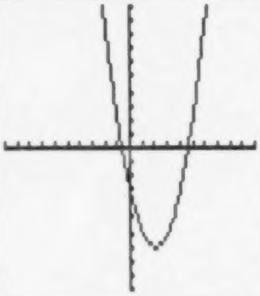
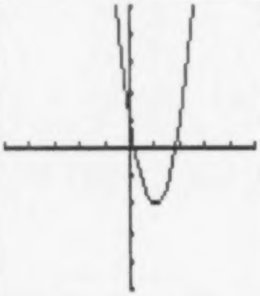
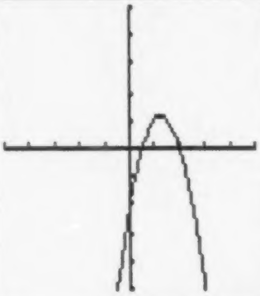
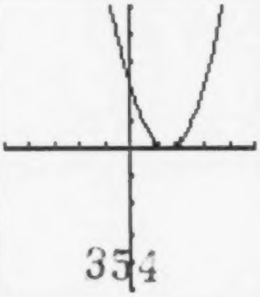
____

Using the blocks or the grids at the bottom of this paper, find a rational number for the square root of each of the numbers listed below. Use your calculator to get the decimal approximation for the rational number. Find a decimal approximation for the square root of each number using the square root function of your calculator. Record your results.

		Rational Square Root	Decimal Approximation	Square Root (Calculator)
1.	200	__________	__________	__________
2.	150	__________	__________	__________
3.	321	__________	__________	__________
4.	247	__________	__________	__________
5.	182	__________	__________	__________

6. Create an algorithm or rule that will generate a rational number that approximates the square root of any whole number.

Function	Solutions from the Quadratic Formula	Graph	Solutions from the Graph
$y = x^2 + 3x - 1$			
$y = x^2 - 4x - 3$			
$y = 3x^2 - 6x + 1$			
$y = -2x^2 + 5x - 2$			
$y = x^2 - 3x + 2.25$			

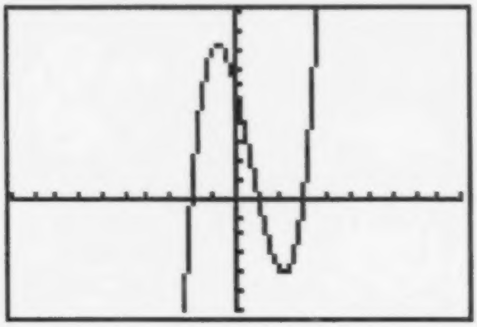
Function	Solutions from the Quadratic Formula	Graph	Solutions from the Graph
$y = x^2 + 3x - 1$	$\frac{-3 \pm \sqrt{13}}{2}$		-3.3, 0.3
$y = x^2 - 4x - 3$	$2 \pm \sqrt{7}$		-0.6, 4.6
$y = 3x^2 - 6x + 1$	$\frac{3 \pm \sqrt{6}}{3}$		0.2, 1.8
$y = -2x^2 + 5x - 2$	2, 1/2		2, 0.5
$y = x^2 - 3x + 2.25$	3/2		1.5

1. Explain how solutions to the quadratic equation, $\mathbf{x^2 - 5x + 1 = 0}$, can be found from a graph.

2. Explain how you can solve $\mathbf{x^2 = 0.1x - 0.3}$ without the quadratic formula.

3. Describe the graph of a quadratic equation which has only one unique solution.

4. What are the solutions to the equation whose graph looks like this:



Bonus: Write an equation that cannot be factored that has two negative solutions. Explain the method you used to get this equation.

Open Boxes

Which box was chosen to be the one with the greatest volume? ______________________

Measure each box that was constructed, record each box's dimensions in the chart below, and calulate and record the volumes of the boxes.

height	length	width	volume
1 cm			
2 cm			
3 cm			

Which box has the largest volume? How does this compare with the visual prediction?

____

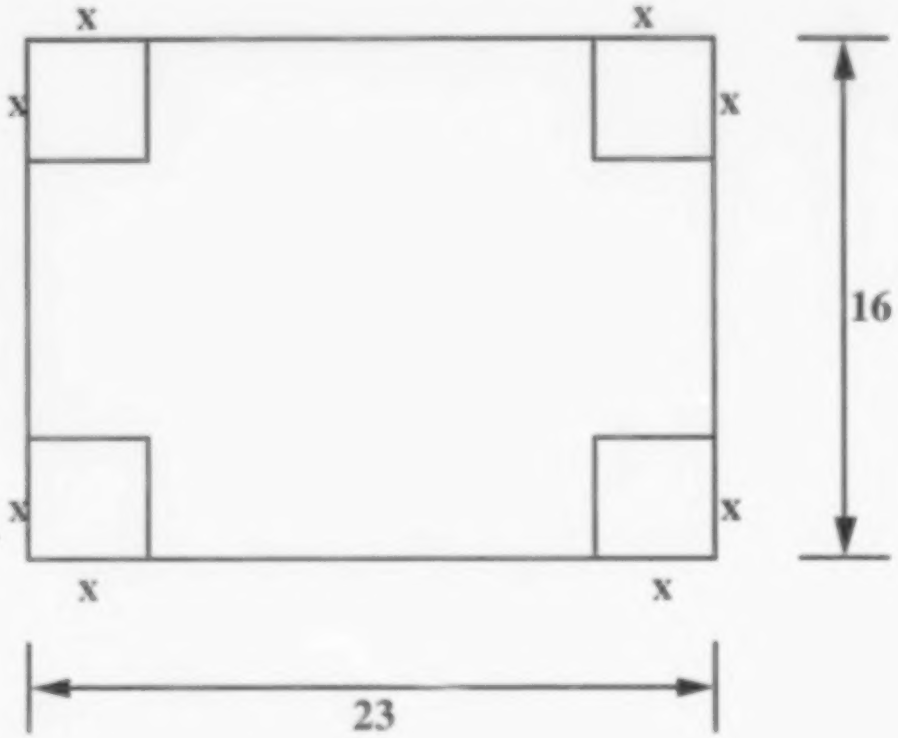
Graph height versus volume. Where is the maximum volume on the graph? ______________

What is the function which describes the volumes of these boxes? ____________________

Graph it and find and record the maximum volume and corresponding dimensions. How does that volume compare with your earlier results?

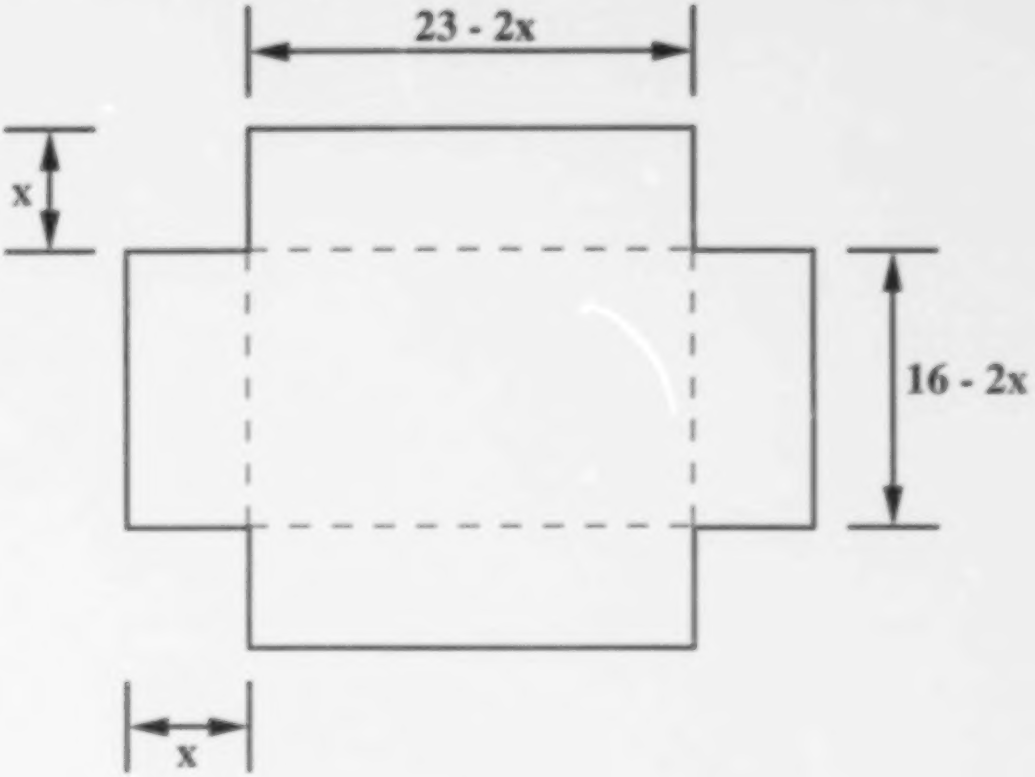
____

Let x represent the length (width) of the squares that are cut out of the corners. As x changes (varies), so do the dimensions of the box.

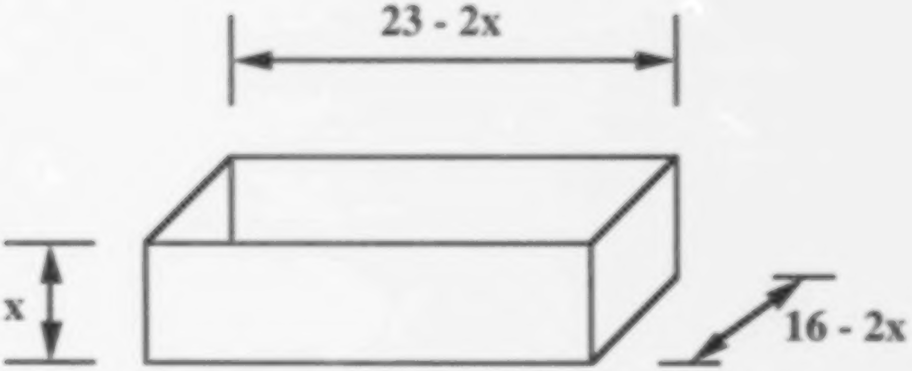


In terms of x, what are the height, length, and width of the box?

In terms of x, what is the volume of the box?


23 - 2x
x
16 - 2x
x

Height = x Length = 23 - 2x Width = 16 - 2x

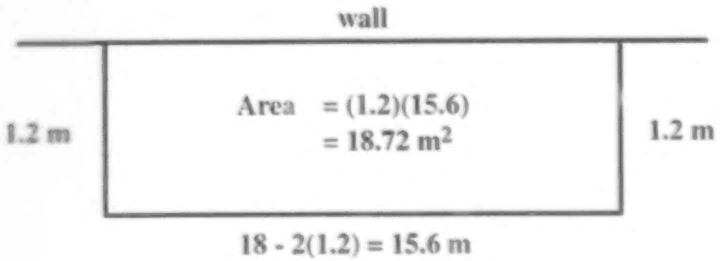


Volume = x • (23 - 2x) • (16 - 2x)

The Maximum Garden

Bernice and Jason are planning a vegetable garden. There exists a wall around their back yard that they will use as one border for the garden. They have enough money to purchase 18 meters of garden fence from Lowe's. The fence sections are predesigned and are available in lengths of 1.2 meter sections or .9 meter sections. In order to have a garden with the maximum area possible, which fence section should they purchase? What would be the dimensions of their garden?

Here is an example of a possible garden:



Use a spreadsheet or calculator to find all possible gardens for both the 1.2 m sections and .9 m sections. What length sections should they purchase to have a garden with the greatest possible area?

1.2 m fence sections

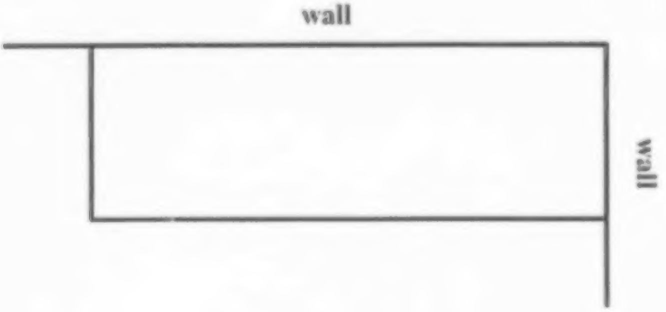
Width	Length	Area

0.9 m fence sections

Width	Length	Area

Graph the width versus the area. What shape is the graph?

Suppose Bernice and Jason decide to plant the garden with two sides of the wall used as sides of their garden.



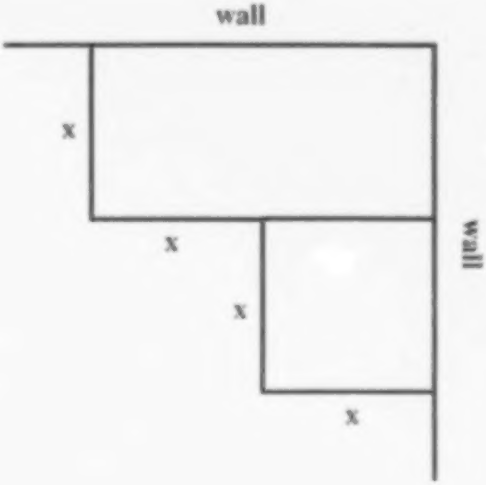
Make a table using a spreadsheet or calculator for each of the widths, 1.2 and 0.9 meters. Which dimensions give the maximum area?

Width	Length	Area

Width	Length	Area

Graph the width versus the area. What shape is the graph?

Bernice thought of another way they could arrange the garden using the wall for two sides, but using only the 0.9 m sections.



Make a table using a spreadsheet or calculator. Which dimensions give the greatest area?

Width	Length	Area

Extension: *The Maximum Garden (A-81)*

Graphing on the TI-82

Tell students to return to the menu **Y=** and remove the equation, **y = 18 - 2x,** in the order to graph only the width versus the area, **y = x(18 - 2x)**. The window can be set to:

Xmin = 0
Xmax = 10
Xscl = 1
Ymin = 0
Ymax = 50
Yscale = 5

From the graph, the students can trace the maximum value of this parabola (4.5, 40.5).

However, this is not the same as the maximum of the garden problem for either the 1.2 or the .9 m fence sections. Ask students, "why not?"

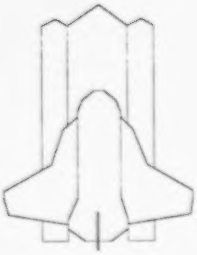
Discuss with them the restrictions on **x** for the fence sections; **x** can only be a multiple of 1.2 (1.2, 2.4, 3.6, 4.8, 5.4 etc.) or multiples of .9 (.9, 1.8, 2.7, 3.6, etc).

Students can trace to the proper ordered pair for the maximum area of the application (4.8, 40.3) or (4.5, 40.50).

Students can continue to investigate maximum area using tables with additional garden situations.

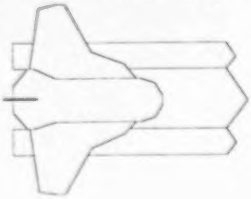
Adapted from:
Channell, D. (1993). Using Calculators to Fill Your Table. In Hirsch, C. and Laing, R. (Eds.) *Activities for Active Learning and Teaching.* Reston, VA: National Council of Teachers of Mathematics.

Shuttle Launch
Recording Sheet



The space shuttle uses solid rocket boosters (SRB) during the launch phase of its flight from Cape Canaveral. The SRBs burn for about two minutes, shut down, detach from the main rocket assembly, and fall back to Earth 140 miles downrange. Parachutes assist the ocean landing, beginning at an altitude of 20,000 feet. The SRBs are recovered and used again for a later launch.

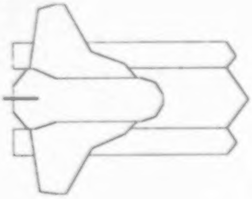
What should the graph of **length of time** the SRBs are in the air **versus** its **altitude** look like?



When the SRBs are on and the shuttle is climbing: $y = 0.002x^2$

When the SRBs are off and detached from the main assembly: $y = -33 + 0.72x - 0.00176x^2$

Graph the system of equations on your calculator and record here.



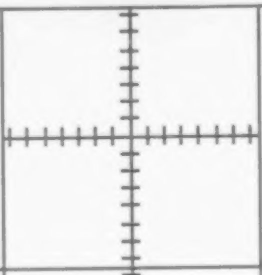
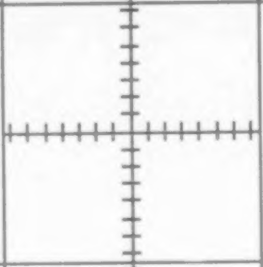
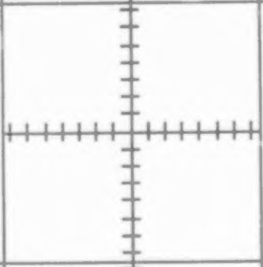
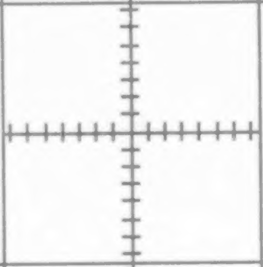
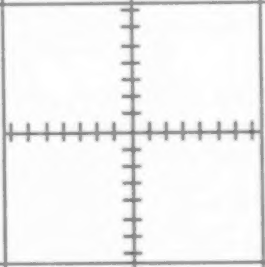
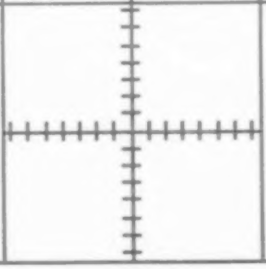
To the nearest second, when did the SRBs cut off?

Determine the maximum altitude attained by the SRBs and at what time did that occur.

How long were the SRBs in the air?

On a separate sheet of graph paper, draw a more detailed graph of the SRBs' flight and identify the critical points of the flight, such as: launch, shut down and separation, maximum altitude, and splashdown. Also identify these points in algebraic terms.

Patterns With Exponential Equations

equation	sketch	y-intercept	increasing or decreasing ?
$y = 1.05^x$			
$y = 3^x$			
$y = 5.25^x$			
$y = 0.3^x$			
$y = 0.75^x$			
$y = 0.15^x$			

Patterns With Exponential Equations

a. For equations of the form $y = b^x$, when $b > 1$, what is the relationship between the value of **b** and the graph of the equation? How do the first three graphs compare?

b. For equations of the form $y = b^x$, when $b < 1$, what happens to the graphs? How do the last three graphs compare?

c. Write an exponential equation that will be increasing.

d. Write an exponential equation that will be decreasing.